Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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Published by The American Society of Mechanical Engineers

July 1949 Vol. 2, No. 7

Applied Mechanics Reviews

Published Monthly by The American Society of Mechanical Engineers
at Easton, Pa., with the co-operation of

THE OFFICE OF NAVAL RESEARCH
ILLINOIS INSTITUTE OF TECHNOLOGY
THE ENGINEERING FOUNDATION
AMERICAN SOCIETY OF CIVIL ENGINEERS
AMERICAN INSTITUTE OF PHYSICS
INSTITUTE OF THE AERONAUTICAL SCIENCES
AMERICAN MATHEMATICAL SOCIETY
SOCIETY FOR EXPERIMENTAL STRESS ANALYSIS
THE INSTITUTION OF MECHANICAL ENGINEERS
THE ENGINEERING INSTITUTE OF CANADA

Editorial Office: APPLIED MECHANICS REVIEWS, Illinois Institute of Technology, Chicago 16, Ill., USA.
Subscription and Production Office: The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., USA.

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ABBREVIATIONS: In abbreviating the titles of periodicals the standard employed in the World List of Scientific Periodicals, Oxford University Press, London, has been followed. In this usage prepositions, articles, and other unimportant words are omitted. Enough of each word is retained to make its meaning obvious, except in the case of common periodical designations such as: J. (Journal); G. (Giornale); C. R. (Comptes Rendus); Z. (Zeitschrift); R. C. (Rendiconti).

Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

APPLIED MECHANICS REVIEWS, July 1949, Vol. 2, No. 7. Published monthly by The American Society of Mechanical Engineers at 20th and Northampton Streets, Easton, Pa., USA. The editorial office is located at the Illinois Institute of Technology, Chicago 16, Ill., USA. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., USA. Cable address "Dynamic," New York. Price $1.50 per copy, $12.50 a year; to members of ASME and co-operating societies $0.75 per copy, $9 a year. Changes of address must be received at Society headquarters three weeks before they are to be effective on the mailing list. Please send old as well as new address. . . . By-Laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publications (B13, Par. 4). . . . Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897. . . .

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Theoretical and Experimental Methods

(See also **Revs. 826, 831, 888, 898, 906, 922, 927, 954**)

814. **V. I. Smirnov, "The life of A. M. Liapounoff"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Sept.-Oct. 1948, vol. 12, pp. 469–478.

A biographical sketch of an outstanding mathematician, A. M. Liapounoff (1857-1918). I. S. Sokolnikoff, USA

815. **V. I. Smirnov, "Survey of the scientific work of A. M. Liapounoff"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Sept.-Oct. 1948, vol. 12, pp. 479–560.

This article contains a brief account of the methods used and the results obtained in the scientific contributions of Liapounoff in connection with the following problems: stability of equilibrium and motion of mechanical systems with a finite number of degrees of freedom; the existence of equilibrium configurations of rotating liquids approximating ellipsoids; stability of equilibrium configurations of rotating fluids; theory of potential and the problem of Dirichlet; theory of probability.

I. S. Sokolnikoff, USA

816. **F. E. Relton, "Applied differential equations,"** Blackie and Sons, London, 1948. Cloth, 8.7 × 6 in., 264 pp., 12 figs., approx. $4.

In the reviewer's opinion this book would make an excellent text for a one- or two-semester course in applications of differential equations, given by engineers to advanced undergraduate engineering students. However the level of the entire book is too low for it to be of use to research workers in mechanics.

R. C. Prim, USA

817. **B. van der Pol and H. Bremmer, "Modern operational calculus based on the two-sided Laplace integral,"** *Indagationes Math.*, 1948, vol. 10: no. 4, pp. 338–345; no. 5, pp. 349–360.

The classical operational calculus, as introduced by Heaviside, finds its rigorous basis in the one-sided Laplace transformation, the transform $f^*(p)$ of the function $h(t)$ being

$$f^*(p) = p \int_0^\infty e^{-pt} h(t)dt.$$

The authors have worked out a new operational calculus, based on the two-sided Laplace transformation,

$$f(p) = p \int_{-\infty}^{+\infty} e^{-pt} h(t)dt$$

denoted by

$$f(p) \doteq h(t).$$

By introducing the unit function $U(t)$

$$U(t) = \begin{cases} 1 \text{ for } t > 0, \\ \frac{1}{2} \text{ for } t = 0, \\ 0 \text{ for } t < 0, \end{cases}$$

the one-sided calculus may be represented as a special case of the two-sided one, according to the formula:

$$p \int_{-\infty}^{+\infty} e^{-pt} h(t)U(t)dt = p \int_0^\infty e^{-pt} h(t)dt.$$

The general advantages of the new operational calculus are the following: (1) a simpler formulation of the general operational rules (such as differentiation, shift rule, composition product); (2) the possibility of treating functions whose two-sided image is simpler than the one-sided image; and (3) the possibility of considering images having no original at all in the one-sided calculus.

But a new difficulty arises from the fact that, while the one-sided Laplace integral has a half plane of convergence, the two-sided one admits only a strip of convergence $\alpha < \mathbf{Re} p < \beta$ so that: (1) the images of two different originals may have nonoverlapping strips and cannot be used together without a new transformation; (2) the same function $f(p)$ may be, for several nonoverlapping different strips, the image of different originals (therefore the indication of the strip must not be omitted).

After giving the rules for the use of Dirac's impulse function, the authors present the following applications of the new operational calculus, in order to show its fertility: (1) linear differential equations with constant coefficients and boundary conditions; (2) linear differential equations with variable coefficients; (3) originals with arguments of exponential character, the study of which leads to the triplet of relations:

$$h(t)U(t) \doteq f(p) \qquad 0 < \mathbf{Re} p < \infty,$$

$$h(e^t) \doteq \frac{1}{\Gamma(p)} \int_0^\infty s^{r-1} f(s)ds \equiv f_1(p),$$

$$f(e^{-t}) \doteq \Gamma(p) f_1(p)$$

which are successfully applied to Bessel, Legendre, and hypergeometric functions; (4) operational identities and generating functions.

The reviewer noticed the following errata: The letter π is often used instead of Γ. The signs of the left members of formulas (7) and (8) should be changed. André Revuz, Turkey

818. **A. Robinson, "On the integration of hyperbolic differential equations,"** *Coll. Aero. Cranfield Rep.*, no. 18, July 1948, 26 pp.

Two methods are considered for the numerical integration of an nth order hyperbolic linear partial differential equation

$$\sum_{k=0}^{n} \sum_{l=0}^{k} a_{kl}(x, y) \frac{\partial^k z}{\partial x^{k-l} \partial y^l} = a_0(x, y) \tag{1}$$

with the initial conditions

$$z(0, y) = g_0(y), \left(\frac{\partial z}{\partial x}\right)_{x=0} = g_1(y), \ldots, \left(\frac{\partial^{n-1} z}{\partial x^{n-1}}\right)_{x=0} = g_{n-1}(y),$$

where $g_i(y)$, $i = 0, \ldots, n-1$ are differentiable $n - i - 1$ times with respect to y. Both procedures are based on the replacement of the above equation by a system of first-order equations of the form

$$\frac{Df_i}{D_i x} = F_i(x, y, f_1, f_2, \ldots f_m), \; i = 1, 2, \ldots, m, \tag{2}$$

where the $f_i = f_i(x, y)$ are a new set of dependent variables and where Df_i/D_1x signifies differentiation of f_i with respect to x along one of the characteristic curves of the system. The functions F_i on the right-hand side only involve the unknown functions, but not their derivatives.

The author first considers the integration of the system of equations (2) by replacing it with its equivalent finite difference representation. However, the discussion of the convergence of this process is left to a later paper. A second procedure, analogous to Picard's method for the construction of the solution of a system of differential equations by successive approximation, which is also based on the system (2), is discussed in detail. Both existence and uniqueness of the solution are demonstrated by the use of this method. H. Polachek, USA

819. R. H. J. Germay, "Remark on a method of successive approximation for the integration of systems of linear differential equations; extension to normal systems of general form (Remarque sur une méthode d'approximations successives pour l'integration des systèmes linéaires d'equations différentielles; extension à des systèmes normaux de forme générale)," *Ann. Soc. sci. Brux. Ser. I*, Dec. 30, 1948, pp. 109–113.

The system of three linear ordinary differential equations,

$$\frac{dy_1}{dx} = f_1(x, y_1, y_2, y_3),$$

$$\frac{dy_2}{dx} = f_2(x, y_1, y_2, y_3),$$

$$\frac{dy_3}{dx} = f_3(x, y_1, y_2, y_3),$$

in four real variables may be solved by means of the recurrence formulas (a modification of Picard's method):

$$y_{1,\mu+1}(x) = y_1^0 + \int_{x_0}^{x} f_1[t;\ y_{1,\mu}(t),\ y_{2,\mu}(t),\ y_{3,\mu}(t)]dt,$$

$$y_{2,\mu+1}(x) = y_2^0 + \int_{x_0}^{x} f_2[t;\ y_{1,\mu+1}(t),\ y_{2,\mu}(t),\ y_{3,\mu}(t)]dt,$$

$$y_{3,\mu+1}(x) = y_3^0 + \int_{x_0}^{x} f_3[t;\ y_{1,\mu+1}(t),\ y_{2,\mu+1}(t),\ y_{3,\mu}(t)]dt,$$

where $y_{j,\mu}$ represents the μth approximation to y_j. (Note the occurrence of $\mu + 1$ on the right-hand side.) The process converges uniformly toward the integral of the system of differential equations. Formulas are given for estimating the error of the μth approximation, in terms of the Lipschitz constants of f_1, f_2, f_3.
R. L. Pigford, USA

820. S. U. Benscoter, "The partitioning of matrices in structural analysis," *J. appl. Mech.*, Dec. 1948, vol. 15, pp. 303–307.

Partitioning of matrices, a procedure now fairly familiar due to the work of Gabriel Kron, is presented by the author in the following form: If $Ax = k$ symbolizes a set of linear equations, all letters denoting matrices (or vectors), and if A is partitioned into four matrices P, Q, R, S according to the scheme

$$\begin{matrix} P & Q \\ R & S, \end{matrix}$$

and if the vectors x and k are appropriately partitioned into (y, z) and (m, n), the following equation for y is obtained:

$$(P - QS^{-1}R)y = m - QS^{-1}n.$$

A similar equation holds for z.

The author applies the method to the following two problems: (1) a thin-walled box beam with corrugated top surface reinforcement; (2) a box-beam section with a sandwich type of top surface. Shear flows and the torsion constant are determined in both cases. A numerical example of problem (2) is given.
P. J. Vlahakis, USA

821. N. N. Bautin, "Criteria for unsafe and safe bounds of a region of stability" (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, Nov.-Dec. 1948, vol. 12, pp. 691–728.

The author treats first the problem of stability and instability of the solutions of the nonlinear differential equation

$$\frac{dx_i}{dt} = \Sigma_{j=1}^{3} a_{ij}x_j + P_i(x_1, x_2, x_3), \quad i = 1, 2, 3$$

where the P_i are power series in the x_i, lacking constant and first-degree terms. Although a reasonably complete theoretical solution to the problem has been given by Liapounoff and his pupils, the algebraic-transcendental problem of determining the stable and unstable regions of coefficient space remains. The simplest regions, corresponding to the case of all the characteristic roots having negative real parts, may be determined quite easily by applying Hurwitz's criteria to the characteristic equation of $A = (a_{ij})$. If there are characteristic roots with zero real parts, the coefficients of the terms in the P_i play an influential role.

The author presents the results of the necessary calculations for these cases and applies the results to the treatment of several electronic circuits. After the discussion of the three-dimensional case, the four-dimensional case is attacked. Here the calculations are truly Herculean in magnitude. Several clear and interesting geometric diagrams supplement the text. R. Bellman, USA

822. P. Y. Poloobarinova-Kochina, "On a nonlinear equation in partial derivatives appearing in filtration theory" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Dec. 1948, vol. 63, pp. 623–626.

This paper is concerned with solutions of the equation

$$\partial^2 u^\mu/\partial x^2 = \partial u/\partial t,\ \mu \geq 1,$$

a type of equation occurring in problems of ground-water flow ($\mu = 2$) and diffusion of gases through porous mediums, with boundary conditions $u(0, t) = A$, $u(x, 0) = B$. As is customary when $\mu = 1$, the author introduces the variable $\xi = x/(2t^{1/2})$ and obtains the equation $d^2u/d\xi^2 = -\xi du/d\xi$. Boundary conditions are taken as $u(0) = 1$, $u'(0) = \alpha$; however for $\alpha > 0$ it is shown that for $\xi \to \infty$ $\lim u(\xi)$ exists so that the previous boundary conditions can be satisfied. For the case $\mu = 2$ solutions are obtained numerically for $\alpha = \pm 0.1, \pm 0.2, \pm 0.3$ and compared graphically with the corresponding solutions when $\mu = 1$. The case $\mu = 2$ is also solved numerically for boundary conditions $u(0) = 0$, $u(\infty) = 1$, and the solution compared with that when $\mu = 1$.
J. V. Wehausen, USA

823. L. E. Grinter (editor), "Numerical methods of analysis in engineering," by Hardy Cross, L. E. Grinter, A. J. Pyka, F. S. Shaw, R. V. Southwell, M. M. Frocht, L. M. K. Boelter, Myron Tribus, G. M. Dusinberre, J. D. Bottorf, Y. S. Touloukian, Frank Baron, N. M. Newmark, and Thomas J. Higgins, MacMillan Book Co., New York, 1949. Cloth, 9.5 × 6.2 in., 207 pp., 89 figs., $5.80.

Much useful information about details of numerical calculations is scattered over the ten articles, by different authors, of this book. A general idea followed in the selection and in the organization of the material is not perceptible, except that a certain

animosity to orthodox mathematical methods breaks through the clouds of intuitive argumentation in almost all contributions.

In chapter 4 R. V. Southwell discusses the accuracy in computations using finite-difference ratios to replace derivatives, and reaches the conclusion that reducing the mesh width is more efficient than increasing the order of the approximation. M. M. Frocht, in chapter 5, offers an interesting suggestion for finding a first approximation for the problem $\Delta u = 0$ with given u-values on the boundary of a convex region. Chapters 6 and 7 deal with simple problems of heat transfer and propose practical solutions based on very rough approximations.

Most other contributions are concerned with the elasticity problems which were the starting point for the numerical procedures known today as "relaxation method." No clear distinction is made between initial-value problems, which permit a step-by-step computation, and boundary-value problems where the essential point is the solution of a large set of simultaneous equations (not directly accessible to computing machines). It should also be made clear to the reader that, no matter in what way the algebraic equations are solved, the main difficulty consists in the admissibility of the transition from differential to difference equations. R. von Mises, USA

824. **Bent Gehlshøj, "Electromechanical and electroacoustical analogies and their use in computations and diagrams of oscillating systems"** (in English), *IngenVidensk. Skr.*, 1947, no. 1, 142 pp.

The author has attempted, in a relatively short space, to present the foundations of the electrical-analogy technique. The material is presented in three chapters entitled "Mechanical Systems," "Acoustical Systems," and "Electromechanical Transducers." Unlike previous treatments of the subject, which are essentially collections of physical systems together with suggested circuit equivalents, fully half of the material presented is devoted to a discussion of fundamentals. As such, it is recommended reading for all serious workers in the field.

It is recognized that there exists a schematic diagram for a mechanical system just as there is a circuit diagram for an electrical network. If then, one constructs a network whose schematic is topologically equivalent to that of the mechanical system, and which is controlled by differential equations of identical form, one is led to a use of the force-current analogy as introduced by Hahnle and Firestone. It is further pointed out that it is possible to deduce a physically realizable network based upon the force-voltage analogy, provided the circuit is planar, by dualizing the original network. The author adds to existing confusion by naming the two analogies, in the order mentioned, Analogy II and Analogy I.

The term impedance is common to the literature in electrical, vibration, and acoustical engineering. This has led the author to attach a similar meaning to the term in all cases and thus to an insistance upon ultimately deducing an equivalent circuit based upon Analogy I.

Recognition is given to the fact that in Newtonian mechanics the acceleration of particles must be referred to an inertial frame of reference. Thus, as a two-terminal device, masses always terminate on such a reference bus. Strange to say, the author is not consistent in this notion, for he objects to Firestone's symbol for a mass (which shows a terminal to ground) and implies that masses can be effectively connected in series since the schematic for a generator placed between two masses looks like a series connection. No mention is made of the fact that a series connection is impossible when three or more masses are present.

The author clearly recognizes the fundamental role of transformers as actually existing in mechanical systems, as the equivalent of a transducer, and as a means of grounding masses. On the other hand, much of the value of this notion is lost by his penchant for eliminating all such elements from his equivalent circuits.

In common with most writers, the author considers that there is a strong analogy between the particle motions in one-dimensional acoustic systems and the motions of electrons in a wire. This leads to the use of Analogy I for treating acoustic systems and also to a lengthy explanation of current continuity when treating a Helmholtz resonator.

The foregoing remarks are made, not to detract from the value of this book which is high, but rather to caution readers that the analogous-circuit technique is still in a controversial state.

Horace M. Trent, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Rev. 829)

825. **"Reissner anniversary volume,"** J. W. Edwards, Ann Arbor, 1949. Cloth, 6.3 × 9.5 in., 493 pp., 147 figs., $6.50.

This volume has been edited by R. P. Harrington, N. J. Hoff, and Paul Torda to commemorate the seventy-fifth birthday of Hans J. Reissner, whose pioneering contributions to propeller theory, airplane structure and strength of materials have made him one of the most important personalities in the field of applied mechanics. In addition to a brief scientific biography of Professor Reissner and a list of his publications (92 entries), it contains five papers in the field of aerodynamics (Bergman, Harrington and Libby, Lew, Riabouchinsky, Tollmien), five papers in dynamics (Goland, Lieber and Hamilton, Oldenburger, Williams, Yuan and Morduchow), twelve in elasticity and structures (Donnell, Friedrichs, Gran Olsson, Eric Reissner, Schleusner, Schnadel, Stoker, Hoff, Salerno, Liebowitz, Boley, Nardo), two in electricity (Foster, Rudenberg), three in mathematical methods (Geiringer, Grammel, Weinstein), three in plasticity (von Mises, Nádái, Odqvist), and two in propulsion (von Kármán, Torda). The total is thirty-two papers, most of which are reviewed separately. L. H. Donnell, USA

✎826. **Richard von Mises and Theodore von Kármán (editors), "Advances in applied mechanics,"** Academic Press, Inc., New York, vol. 1, 1948. Cloth, 9.3 × 6 in., 293 pp., 63 figs.

This volume contains the following articles:

H. L. Dryden, "Recent advances in the mechanics of boundary layer flow," 42 pp. [see Rev. **1517**, Oct. 1948].

N. Minorsky, "Modern trends in nonlinear mechanics," 64 pp. [see Rev. **1550**, Oct. 1948].

C. B. Biezeno, "Survey of papers on elasticity published in Holland 1940-46," 63 pp. [see Rev. **1600**, Nov. 1948].

J. M. Burgers, "A mathematical model illustrating the theory of turbulence," 31 pp. [see Rev. **775**, June 1949].

Hilda Geiringer, "On numerical methods in wave interaction problems," 47 pp. [see Rev. **508**, Apr. 1949].

R. von Mises and M. Schiffer, "On Bergman's integration method in two-dimensional compressible flow," 38 pp. [see Rev. **1506**, Oct. 1948]. L. H. Donnell, USA

827. **H. I. Andrews, "The mobile locomotive testing plant of the London, Midland and Scottish Railway,"** *Proc. Instn. mech. Engrs.*, 1948, vol. 158, no. 4, pp. 450–476.

A special testing train has been constructed in which the drawbar pull of the locomotive is absorbed by a number of electrically braked cars, the loading of which is controlled by an electronic regulator so that constant speed is automatically maintained and

the effects of gradients, curves, etc., are immediately compensated. The train consists of three braking units (which are, in effect, powerful motor coaches equipped for rheostatic braking only), a dynamometer car, and a special tender for measuring the fuel and water consumption of steam locomotives.

A. R. Holm, Denmark

828. W. K. G. Floor, "Reliability of the drop weight reduction method for simulating wing lift effect in landing gear drop tests" (in English), *Nat. LuchtLab. Amsterdam Rap.*, no. S.340, July 30, 1947, 16 pp., 8 tab., 16 figs.

This paper deals with drop tests of aircraft landing gears. The effect of wing lift in reducing the load upon the landing gear in actual aircraft is simulated by employing a reduced weight and utilizing a free drop. The maximum force transmitted by the landing gear is investigated for the actual and simulated conditions by analyzing a damped mass-spring system. It is shown that the difference in maximum transmitted force for the two conditions is a few per cent, provided the efficiencies of the landing gear are equivalent. Efficiency e is defined as $E/(Pd)$ where E is the energy absorbed, d the maximum deflection, and P the maximum transmitted force. The simulated test gives lower maximum forces than the actual condition.

Charles E. Crede, USA

Gyroscopics, Governors, Servos

(See also Rev. 913)

829. Tadao Miki, "On the Raumkompass" (in Japanese), *Appl. Math. Mech. (Ōyō Sūgaku Rikigaku)*, Oct. 1947, vol. 1, pp. 202–210.

The motion of the Anschütz *Raumkompass*, which is a compass involving two equal gyroscopes connected by a spring, is investigated analytically. It is shown that it is free from any error due to the acceleration (including rolling and pitching) of the ship on which it is mounted. The stability of the north-pointing property is confirmed by the method of small oscillations.

Isao Imai, Japan

Vibrations, Balancing

(See also Revs. 824, 829, 874, 925, 928)

830. Antonio Ruelle, "Critical conditions of a railway track under a spring-born load (Condizioni critiche del binario percorso da un carico molleggiato)," *Atti. Ist. Sci. Costr. Univ. Pisa*, 1948, no. 6, 9 pp.

The paper is an extension of S. Timoshenko's solution of the dynamic problem of a railway track. The effect of a spring-born load and its influence on critical conditions of the rail are considered.

Enrico Volterra, USA

831. Béla de Sz. Nagy, "Vibrations of a nonhomogeneous string (Vibrations d'une corde non homogène)," *Bull. Soc. math. Fr.*, 1947, vol. 75, no. 1-4, pp. 193–208.

This treatment of the vibrations of a nonhomogeneous string utilizes the spectral theory of completely continuous self-adjoint operators in Hilbert space. Standard results on the nature of the normal vibrations and the expansion problem for general vibrations are extended to the general case of an increasing mass function. The final motivation for the abstract-operator technique, however, is the application of the perturbation theory of self-adjoint operators developed by Rellich and extended by the author [*Comment. Math. Helv.*, 1947, vol. 19, pp. 347–366] to obtain explicit estimates for the rapidity of convergence of the (Rayleigh) perturbation series corresponding to a small change in mass function. The case of a homogeneous string with a point load is worked out in detail.

Courtesy of Mathematical Reviews W. F. Eberlein, USA

832. Teruyosi Udoguti, "On the relation between the whirling and lateral vibrations of a rotating shaft" (in Japanese), *Trans. Soc. mech. Engrs. Japan*, 1948, vol. 14, no. 47, part 1, pp. 75–83.

This paper deals with the lateral vibrations of rotating shafts, under boundary conditions varying with their speeds of rotation. As an example, the author calculates the frequency of vibration of a cantilever shaft loaded with a disk at its free end. While its natural frequency decreases with increase of the moment of inertia of the disk, its critical speed increases. This tendency becomes greater for higher modes of vibration. The results of calculations for a shaft supported freely at both ends are also given.

Sumiji Fujii, Japan

833. A. Okumura and M. Ikeda, "The effect of an elastic clamp on the frequency of a rod vibrating transversely" (in Japanese), *Trans. Soc. mech. Engrs. Japan*, 1948, vol. 14, no. 46, pp. 22–31.

This paper deals with the influence of an elastic clamp on the frequencies of a vibrating rod. Numerical tables and formulas are given for various combinations of clamping conditions (that is, of the "softness" for angular and transverse displacements of the ends) and modes of vibrations. With increase of the softness at the clamped ends the frequency becomes lower, and this tendency increases for the higher modes.

Sumiji Fujii, Japan

834. Y. Sawaragi, "On forced vibrations of systems with restoring forces of certain types" (in Japanese), *Trans. Soc. mech. Engrs. Japan*, 1947, vol. 13, no. 45, pp. 70–80.

A forced vibration represented by the relation

$$m\ddot{x} + f(x) = P \sin \omega t,$$

where $f(x)$ has a discontinuity at $x = 0$, is discussed by expanding x and $f(x)$ into a Fourier series in t. The results are compared with those obtained by Martienssen's method and two other methods of approximation. Various accurately calculated resonance curves are given.

Sumiji Fujii, Japan

835. Sumiji Fujii, "On periodic slip motion on a rough surface" (in Japanese), *Appl. Math. Mech. (Ōyō Sūgaku Rikigaku)*, Nov.-Dec. 1947, vol. 1, pp. 278–287.

A mass is placed on a plate and attached to a fixed point by a spring. When the plate is moved with a constant velocity "stick-slip" motion occurs, if the coefficient of friction varies with the relative motion of the mass to the plate. The author discusses these problems using the phase plane. It is concluded that four different types of self-excited oscillations can take place, according to the mode of variation of the friction with the relative velocity.

Takeo Mogami, Japan

836. T. Takahashi, "On a quasi-harmonic vibration" (in Japanese), *Trans. Soc. mech. Engrs. Japan*, 1948, vol. 14, no. 46, pp. 1–7.

Vibration with a variable restoring coefficient of the type $m\ddot{x} + 2\epsilon\dot{x} + f(t)x = 0$, where $f(t) = ke^{2\alpha t}$, is dealt with. Putting $\tau = e^{\alpha t}$,

the equation is transformed into a differential equation of the Bessel type, the solution of which is given in the form:

$x = Ae^{-\epsilon t/m} J_\mu(\beta e^{\alpha t}) + Be^{-\epsilon t/m} Y_\mu(\beta e^{\alpha t})$, where $\beta^2 = k/m\alpha^2$ and $\mu = \epsilon/m\alpha$. The paper also treats the problem of the vibration of a string with variable tension given by

$$\rho \partial^2 y/\partial t^2 = T(t)\partial^2 y/\partial x^2.$$

Putting $y = u(x)v(t)$, it is shown that an equation of the same type as the former is obtained. Further, a solution for the vibration of a bar with variable bending rigidity is described.

Sumiji Fujii, Japan

837. **J. M. Burgers, "Damped oscillations of a spherical mass of an elastic fluid"** (in English), *Proc. kon. Ned. Akad. Wet.*, Dec. 1948, vol. 51, pp. 1211–1221.

A theoretical treatment of the oscillations of certain soap solutions described by B. de Jong [*Proc. kon. Ned. Akad. Wet.*, 1948, vol. 51, pp. 1197–1210] is carried out for a spherical mass of the fluid moving (*a*) in concentric spherical layers and (*b*) axisymmetrically in meridian planes. Three separate cases are considered: damping through viscous forces, through relaxation of elastic forces, and through slipping of a fluid relative to the wall of its container. In these three cases and in the order listed above, the logarithmic decrement of the amplitude of the oscillation is found to vary inversely as the radius of the container, directly proportional to the radius, and independent of the radius. The period of oscillation which is found in the various cases is always proportional to the radius. Numerical data concerning the shear modulus, the relaxation time, and the coefficient for the slipping of a fluid against a wall will be given by de Jong in forthcoming papers.

Edward Saibel, USA

Wave Motion, Impact

(**See also Revs. 828, 842, 925, 926, 953**)

838. **W. R. Dean, "On the reflexion of surface waves by a submerged circular cylinder,"** *Proc. Camb. phil. Soc.*, Oct. 1948, vol. 44, pp. 483–491.

The author analyzes the problem of the reflection of two-dimensional surface waves of small amplitude in a liquid in which a circular cylinder is submerged. The result of the paper is that the coefficient of reflection, or the ratio of the amplitude of the reflected wave to that of the incident wave, is zero. A series expansion for the velocity potential is found, which formally satisfies the boundary condition at the free surface. The coefficients in this expansion are to be determined by satisfying the boundary condition at the surface of the cylinder. This boundary condition leads to a sequence of algebraic equations for the coefficients of the potential expansion. One particular case is worked out numerically, and the first few coefficients are evaluated.

The general conclusion that the reflection coefficient is zero is not affected by the fact that the solution must be determined numerically for any set of values of the parameters. The author's result of zero reflection coefficient is independent of the depth of the cylinder below the surface. That this conclusion is valid for very small submergence must be a consequence of the linearization of the wave problem.

Milton S. Plesset, USA

839. **Merit P. White, "On the impact behavior of a material with a yield point,"** *J. appl. Mech.*, Mar. 1949, vol. 16, pp. 39–54.

The test data obtained by D. S. Clark and P. E. Duwez at the California Institute of Technology on the longitudinal impact of specimens of an iron and a steel with definite yield points havstimulated an analysis by the author.

A brief review of some of the published theories and analyses of plastic impact are given by way of introduction. Idealized stresse strain curves are described for materials with a definite yield point, and for conditions in which these stress-strain curves may be independent or dependent on strain rate. The possible kinds of behavior of longitudinal waves, both elastic and plastic, are explained through the use of "position-time" diagrams for the two types of idealized stress-strain curves. Data on the "lower" and "upper" actual velocities are reviewed and compared with values calculated from equations derived from the author's analysis. The same is done for the strain distributions. Because of the complex nature of the reflections of the plastic and elastic waves these latter calculations are difficult. The author uses position-time diagrams to make his calculations and assumptions clearer to the reader.

The author claims that his analysis indicates that a material with a definite yield point ruptures under impact at a higher (engineering) stress and smaller strain than when loaded statically. The reviewer is unable to see, however, that this is proven by the author's analysis.

M. J. Manjoine, USA

Elasticity Theory

(**See also Revs. 847, 848, 879, 880, 881**)

840. **E. Orowan, "Classification and nomenclature of internal stresses,"** *Inst. Metals Monogr. Rep. Ser.*, 1948, no. 5, pp. 47–59.

The author reviews the main types of internal stresses, and classifies them into two groups according to the manner in which they originated: (1) body stresses, arising from nonuniform external (mechanical, thermal or chemical) influences acting upon the body; (2) textural stresses, due to textural inhomogeneities which may be present in the material initially, or produced by plastic deformation or structural changes. The main purpose of the article is to emphasize the variety of different phenomena lumped together under the collective name of internal stresses, with a suggestion of the possibility of investigating textural stresses by means of polarized light.

H. R. Neifert, USA

841. **R. S. Rivlin, "A uniqueness theorem in the theory of highly-elastic materials,"** *Proc. Camb. phil. Soc.*, Oct. 1948, vol. 44, pp. 595–597.

The stored-energy function of an incompressible neo-Hookean material [see the author's previous paper, *Phil. Trans. roy. Soc. Lond. Ser. A*, 1948, vol. 240, p. 459] is here assumed to be of the form $W = C_1(I_1 - 3) + C_2(I_2 - 3)$, where C_1 and C_2 are positive constants characterizing the material and

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \quad I_2 = \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 + \lambda_1^2\lambda_2^2,$$

the λ's being the principal extensions at the point considered. It is proved that, if a unit cube of the material is subjected to the action of equal and oppositely directed forces acting normally on its faces, the finite deformation produced is uniquely determined, provided that the forces per unit area, measured in the deformed state, are specified. It is further proved that the deformed state is stable.

Courtesy of Mathematical Reviews J. L. Synge, USA

842. **M. I. Rozovski, "Impact of a cylinder against the surface of a medium, whose mechanical properties change with time"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, July 1, 1948, vol. 61, pp. 25–28.

The author examines the case of an absolutely rigid right circular cylinder which strikes with its base the surface of a medium whose mechanical properties change with time. He gives a general solution for the components of stress and of deformation, and also the reaction of the medium on the cylinder. For a special case he also gives the results of integration of the basic equations and develops specific formulas for the deformation and for the reaction of the medium. Z. Bažant, Czechoslovakia

Experimental Stress Analysis

(See also Revs. 827, 840)

843. Raymond D. Mindlin, "A mathematical theory of photo-viscoelasticity," *J. appl. Phys.*, Feb. 1949, vol. 20, pp. 206–216.

Until now no great progress has been attained in applying optical methods to the investigation of stresses which are not elastic but are more or less plastic; it is therefore gratifying that, in addition to purely experimental techniques progress is also being made in theoretical work in this field, as exemplified by the present paper.

To simplify the theory, it is assumed that the dependent birefringence, stress and strain have linear relations with one another. General equations are assumed for the dependence between strains and birefringence and also for the relations between stress and strain for an idealized material in which elasticity as well as retarded elasticity and flow is possible. The general strain-optical equation is applied to this material as a special case.

The relations between stress and strain and the optical effects which can be measured most easily are discussed, and examination is made of the conditions under which the results can be represented as in photoelasticity. Simply connected bodies as well as multiply connected bodies are considered. Application of these results as an extension of photoelasticity is discussed.

Ludwig Föppl, Germany

844. R. E. Glover, O. J. Olsen, and Carl Zangar, "Experimental aids in structural concrete design," *J. Amer. Concr. Inst.*, Feb. 1949, vol. 20, pp. 445–466.

The paper presents a comprehensive review of applications of various experimental methods for stress determination, with particular reference to concrete structures and models of such structures. The methods discussed are those employing photoelasticity, the Begg's Deformeter, Stresscoat and electric resistance strain gages. Of particular interest is the description and operation of the interferometer used by the Bureau of Reclamation for photoelastic studies. By means of this instrument, believed to be the only one in operation, the magnitudes and directions of principal stresses may be determined directly.

Morton B. Millenson, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 820, 830, 859, 862, 883, 954)

845. O. Föppl, "Residual stresses in rods of circular cross section under the influence of surface compression (Die Eigenspannungen in oberflächengedrückten Stäben von Kreisquerschnitt)," *Z. Ver. dtsch. Ing.*, Dec. 1948, vol. 90, pp. 369–372.

An analysis is presented to determine the distribution of residual stresses in bars of circular cross section when plastically deformed by surface rolling. The bar is considered to be composed of an elastic core and a thin plastic shell, producing uniform circumferential and axial tensile stresses in the core and uniform compressive stresses in the shell. It is then shown from equilibrium and compatibility conditions that if the bar is bored centrally the resulting strain at the surface varies hyperbolically with the size of the bore. Experimental results seem to substantiate this analysis. Louis F. Coffin, Jr., USA

Plates, Disks, Shells, Membranes

(See also Revs. 853, 854, 861, 862)

846. Lidia Stankiewicz, "Calculation of the deformation of a plate on an elastic foundation (Sul calcolo della deformazione della piastra poggiata su suolo elastico)," *R. C. Accad. Lincei*, Dec. 1948, ser. 8, vol. 5, sem. 2, pp. 339–344.

The paper concerns an infinite plate on an elastic foundation the reaction of which is proportional to the local deflection (Winkler's law). E. Volterra [*R. C. Accad. Lincei*, 1947, vol. 2, p. 595] had reduced this problem to the numerical computation of the integral

$$F(x, y) = \int_0^\infty \int_0^\infty \frac{\sin xu \sin yv}{(u^2 + v^2)^2 + 1} \cdot \frac{du\, dv}{u\, v}.$$

The author shows how the numerical work can best be carried out and gives a table of the results. W. Flügge, USA

847. A. Meldahl, "Temperature stresses in gas turbine rotors at starting" (in English), *Brown Boveri Rev.*, Sept.-Oct. 1948, vol. 35, pp. 247–252.

Known solutions for transient symmetric temperature distributions in solid and hollow rotors are substituted into the conventional formulas for temperature stresses in cylinders. The significant stresses are calculated for the practically important case in which the ambient temperature rises linearly to a terminal value and then remains constant. Dimensionless numerical results are given in graphical form.

Two conclusions of practical interest are: (*a*) that under certain conditions the maximum stresses occur considerably later than the end of the warm-up period, and (*b*) that warm-up periods must be made quite long to reduce the resulting temperature stresses markedly.

Recommendations for numerical values of allowable stresses, heat-transfer coefficients, or other parameters of the problem for steam- or gas-turbine installations lie outside the scope of this paper and are not included. Such information would be necessary to apply the results obtained to particular design problems.

G. A. Nothmann, USA

848. W. Flügge, "On the membrane theory of shells of revolution with negative curvature (Zur Membrantheorie der Drehschalen negativer Krümmung)," *Z. angew. Math. Mech.*, June 1947, vol. 27, pp. 65–70.

The author discusses solutions of the differential equations relating the membrane internal forces in shells of revolution with negative curvature. These equations have real characteristics and are essentially different from those of dome-shaped shells of positive curvature. The mathematical treatment is simple but great stress is laid by the author on the necessity of careful research in this field.

Denoting by N_φ and N_θ the normal forces acting on an element of the shell, by φ and θ angular coordinates in the meridian and transverse planes respectively, and by r_1 and r_2 the radii of curvature, the author shows that the expression

$$\frac{\partial^2 N_\varphi}{\partial \varphi^2} + \frac{r_1}{r_2 \sin^2 \varphi} \cdot \frac{\partial^2 N_\varphi}{\partial \theta^2}$$

is of the elliptic or hyperbolic type for positive or negative curvature respectively. Hence for positive curvature the discontinuities of the boundary loading are equalized and do not penetrate the shell, as indicated by solutions for domes on isolated supports. On the other hand for negative curvature the equations have real characteristics, that is, there exist paths on the shell along which the discontinuities of boundary loading are spread over the entire shell surface. As an example the author discusses the hyperboloid of revolution of one sheet supported on isolated bearers at the lower boundary, such as the reinforced-concrete shells used in the construction of cooling towers.

After deriving certain geometrical properties, the author proves that for isolated boundary loads the membrane theory yields a physically impossible solution, and hence for the solution of such problems a knowledge of the deformation and application of the bending theory of these shells is necessary.

The author then finds the solution for continuous meridional loading and proves that, while for dome-shaped shells the ratio of the tangential and normal loading of the boundary has a fixed value determined by the geometry of the shell, shells of revolution with negative curvature have generally no such restriction. For certain shapes of the shell and loadings, however, the tangential forces at the support may become very large. This phenomenon has practical significance in the design of such supports.

In conclusion the author points to the need for further investigations in the deformation and bending theory of thin shells with nonsymmetric boundary loading. Nicholas Sag, Australia

849. **R. J. Duffin, "On a question of Hadamard concerning super-biharmonic functions,"** *J. Math. Phys.*, Jan. 1949, vol. 27, pp. 253–258.

Hadamard, in his prize memoir on the equation of the clamped plate [*Mémoires présentés par divers Savants étrangers à l'Académie des Sciences*, 1908, vol. 33, no. 4] had raised the following question: "If a perpendicular force is applied at some point of a thin flat elastic plate which is rigidly clamped on its boundary, will the displacement of the plate be of one sign at all points?"

The author considers this question for rectangular plates. He first examines the case of the infinite strip, for which he gives an explicit solution by using the Fourier-transform method. He then examines a finite strip which is approximately rectangular in shape. In this case a solution is not obtained explicitly, but it is shown that if pressure is applied to the center of an approximately rectangular plate four times as long as it is wide, the deflection of this plate at a certain distance from the center will be alternatively positive and negative. On the other hand the author conjectures that, for a square plate, the answer to Hadamard's question is affirmative. This conjecture gains some support from the fact that the answer is affirmative for the circular plate. M. Bricas, Greece

850. **Folke K. G. Odqvist, "Plasticity applied to the theory of thin shells and pressure vessels,"** *Reissner Anniv. Vol.*, J. W. Edwards, Ann Arbor, 1948, pp. 449–460.

Certain methods of plasticity are applied to thin-walled pressure vessels. The type of vessel treated is one found frequently in the chemical industries, especially breweries. It consists of a cylindrical part and two attached dished heads. These heads are comparatively shallow, with a very small radius in the inside corner, and thus can be assumed to be parts of a spherical shell. This and other approximations used in the treatment, together with the assumed limitations, are carefully stated as they occur.

The stresses in the vessel are treated by first assuming Hooke's law to hold throughout. The equations obtained involve the internal pressure and are of course valid for conditions of elastic behavior. However, they are put in such a form that the effect of plastic action on the magnitude of the stresses can be studied. The material of which the vessel is made is assumed to be ideally plastic, that is the stress remains constant after yielding occurs. Similarity in treatment of the thin-walled vessel to the treatment of a beam in bending leads to the conclusion that the vessel, when stressed beyond the elastic limit, will develop a "plastic hinge" (a property also introduced in limit design). This condition is shown to be beneficial in enabling this type of vessel to withstand repeated stressing with comparative safety.

A. N. Zamboky, USA

851. **Alfred John Sutton Pippard and Letitia Chitty, "Experiments on the plastic failure of cylindrical shells,"** *Civil Engineer in War, vol. 3, Instn. civ. Engrs.*, London, 1948, pp. 2–29, 6 plates.

The paper describes a series of tests on the bending of stiffened and unstiffened cylindrical shells, when loaded to failure by radial static and impact loads and supported over half of their surface opposite the load. Additional tests were made under external hydrostatic loading, with similar support.

The tests indicated that the stiffness of the end plates has a marked effect on the deformation. If the stiffness of the end plate is negligible, a transverse groove is formed under a concentrated load and flats develop. Under hydrostatic load the shells failed by development of a single dent, but when the end plates could resist bending out of their original plane failure was multilobed.

Comparison of the energy absorption for static and dynamic failure of steel specimens indicated an important effect of the time factor. Tests on copper specimens did not show similar effects of time. H. H. Bleich, USA

Buckling Problems

(See also Rev. 851)

852. **Paul Seide and John F. Eppler, "The buckling of parallel simply supported tension and compression members connected by elastic deflectional springs,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1823, Feb. 1949, 18 pp.

As an approach to the buckling under bending of box beams with transverse ribs of finite stiffness wherein the additional support of the tension surface must be considered, an energy solution is presented for the buckling in its own plane of a system consisting of a simply supported compression member and a simply supported tension member interconnected by identical and equally spaced springs. Using the results of this solution, charts are drawn showing the critical longitudinal loads for spans of two, three, four, and an infinite number of bays, when the tension and compression loads are equal. John E. Goldberg, USA

853. **W. Flügge, "Determination of the optimum dimensions for sandwich plates (Détermination des dimensions optima des plaques-sandwichs),"** *Rech. aéro. Paris*, Jan.-Feb. 1949, no. 7, pp. 43–49.

The paper contains diagrams for determining the optimum dimensions of infinitely long or infinitely wide flat sandwich panels under uniaxial compression, given the material of the faces and assuming a given constant ratio between the modulus of rigidity and the specific weight of the core material. The materials of faces and core are considered to be isotropic. The use of the diagrams is briefly explained, but for their derivation reference is made to unpublished reports of the Office National d'Études et de Recherches Aéronautiques.

An examination by the reviewer of the results following from the diagrams reveals that the buckling loads for elastic over-all instability are smaller than those obtained by others, for instance as given in the reviewer's paper at the 1948 International Congress of Applied Mechanics. The difference increases for decreasing shear stiffness of the core and decreasing ratio of core thickness to face thickness; it is probably caused by the simplifying assumptions introduced by the author and in practical cases may reach a value of 20 per cent.

Inelastic buckling is evidently assumed to occur when the compressive stress reaches the elastic limit of the face material and the buckling stress is assumed equal to this limit; this appears to be an oversimplification of the problem.

F. J. Plantema, Holland

854. J. M. Klitchieff, "On the stability of plates reinforced by ribs," *J. appl. Mech.*, Mar. 1949, vol. 16, pp. 74–76.

The elastic stability of simply supported rectangular plates having ribs placed transversely to the direction of the axial compressive loads is considered. For the case of equally spaced ribs an expression is developed for the bending rigidity of the ribs required to produce nodal lines at the ribs. The effective widths of the plate acting with the ribs in bending are taken into account.

Application is made to a part of the deck plating of a cargo ship. It is indicated that the strength required according to Lloyd's rules is inadequate. M. V. Barton, USA

855. F. W. David, "Joints in flat panels subjected to shear and combined shear and tension," *Aircr. Engng.*, Apr. 1947, vol. 19, pp. 118–124.

This paper deals with aircraft panels subjected to pure shear as well as to combined shear and tension loads. It is considered that a panel subjected to an increasing shear load develops equal principal tensile and compressive stresses up to the load at which the panel begins to buckle. Further increase in the shear load then causes an increase in the tensile stress and a smaller increase in the compressive stress. Such a stress pattern is normally known as an incomplete tensile field.

The paper presents formulas and charts for determining stresses in the mid-plane of a sheet panel and the loads acting on rivets used to attach stiffeners and flanges bounding the panel. Charts are presented for determining suitable stiffener spacing and the resulting buckling stress of a panel. The results are expressed in nondimensional form. Numerical examples are given to illustrate the use of these charts. Frank Baron, USA

856. Norris F. Dow, "Design charts for longitudinally stiffened wing compression panels," *Soc. auto. Engrs. quart. Trans.*, Jan. 1949, vol. 3, pp. 122–144.

A review of the development of design charts for stiffened compression panels and the experimental work associated with this development conducted by the NACA is presented in this paper. Panels constructed of 24S-T and 75S-T aluminum alloys with formed Z- and hat-section and extruded Y-section stiffeners were studied.

The major conclusions of the project are embodied in design charts, which permit the direct selection of panel proportions to meet the assumed major design conditions of intensity of loading, skin thickness and effective length of panel. Also drawn from this work are the conclusions that a nonbuckling skin is compatible with high structural efficiency, and that ideally proportioned Y-stiffened panels showed higher structural efficiencies than corresponding Z-stiffened panels. Hat-section stiffeners were not found to be more efficient than Z-section stiffeners.

It was noted that "very large diameter rivets very closely spaced are required to achieve a panel strength comparable to that for a panel having integral sheet and stiffeners." Since the design charts are based on such construction, it appears to the reviewer that the lack of a method for evaluating fastener designs which do not correspond to these rigid requirements detracts from the usefulness of the design charts. George Gerard, USA

857. E. E. Lundquist, E. Z. Stowell, and E. H. Schuette, "Principles of moment distribution applied to stability of structures composed of bars or plates," *Nat. adv. Comm. Aero. Rep.*, no. 809, 1945 (issued in 1948), 14 pp.

This is an extension of the Hardy Cross moment-distribution procedure to the problem of determination of the stability of structures composed of long plates under longitudinal loading. Moment-distribution procedures for studying the stability of structures composed of bars were first described by the senior author in 1937 [E. E. Lundquist, "Stability of structural members under axial load," *Nat. adv. Comm. Aero. tech. Note*, no. 617, 1937]. The present report gives generalized derivations applicable to both bar and plate structures.

Both the stiffness and the series criteria for stability are discussed. Examples are given of the application of the methods to a continuous bar and to the local instability of a Z-section column. Formulas are given for the necessary quantities, but no numerical values are given in the report. N. M. Newmark, USA

858. George Winter and R. H. J. Pian, "Crushing strength of thin steel webs," *Cornell Univ. engng. Exp. Sta. Bull.*, no. 35, part I, Apr. 1946 (issued in 1947), pp. 3–24.

Results of 136 tests were used to derive an empirical formula for the crushing strength of webs of light-gage cold-formed I-beams under loads in the plane of the web and normal to the length of the beam. Tests were made on beams 4 in., 6 in. and 8 in. deep formed of 0.046 to 0.473-in. thick steel sheet, having yield strengths S_{yp} of from 30,000 to 38,000 psi. The load P was distributed over lengths b ranging from 0.8 in. to 3.5 in. The data are approximately consistent with the following formula for the crushing load P in pounds: $P=[A+B(b/t)^{1/2}]\,t^2S_{yp}$ where A and B are constants having the values 15 and 3.25 respectively if the loading is at an interior point along the length of the span, and values 10 and 1.25 respectively if the loading is at the ends of the span. For the proportions investigated the load appears to be largely independent of the depth of the flange and of whether the reaction to the load is immediately beneath it or elsewhere.

John E. Goldberg, USA

Joints and Joining Methods

(See Rev. 855)

Structures

(See also Revs. 820, 844, 850, 851, 852, 856, 857, 932)

859. C. V. Klouček, "Distribution of deformation, a new method of structural analysis," Prague, 1949. Paper, 6.5 × 9.5 in., 510 pp., approx. 400 figs.

This book of 510 pages published by the author in English is a translation of an earlier edition published in 1940. In entitling the book *A New Method of Structural Analysis* the author seems to have been unaware of publications in America by Maney, Goldberg, and Grinter each of which antedated his work by several years. For example, he incorrectly states in his conclusion, "The

method of solution of statically indeterminate structures by deformation distribution was not previously known."

The method described is fundamentally the determination by distribution of the usual slope-deflection redundants. Two procedures, "direct distribution" and "successive distribution," are mentioned and large numbers of special cases are investigated. The uniqueness of the work lies in the detailed treatment of these special cases, since the basic principles are described in American literature. The author gives his attention almost exclusively to direct distribution, neglecting successive distribution which has found greater favor in the United States. L. E. Grinter, USA

860. **Edward L. Williams and Percy A. Badland, "The effect of lateral loads on steel-framed buildings,"** *J. Instn. civ. Engrs.*, Dec. 1948, vol. 31, pp. 117–155.

The authors carry out an investigation of the safe wind moments to be resisted by a "rigid-frame" steel structure, starting from a unit consisting of one bay and one story and then proceeding to a multiple-bay, multiple-story structure. The paper includes the effect of semirigid connections (the moment-angle relationship being assumed to be nonlinear) and of the moments caused by the direct column loads due to lateral deflections. This latter effect is commonly ignored in wind-bracing design, but it is shown to result in a substantial reduction of the effective wind-resisting strength. On the other hand, in buildings of medium height, a surplus of strength in the upper stories is shown to assist the first story in resisting lateral loads.

No account is taken of the effect of column shortening and lengthening, which becomes important only in very tall buildings. The designer is assisted by a set of tables and graphs and various design short cuts recommended by the authors.

Ernest F. Masur, USA

861. **Srecko Kraus, "Contribution to the solution of the problem of transversely loaded crossed ribs (Contributo al problema dei solai con nervature incrociate),"** *G. Gen. civ.*, Nov. 1948, vol. 86, pp. 614–617.

This paper considers a system of bars lying in a plane and covering a square area. The bars are equally spaced and extend in the directions of the diagonals of the square; they therefore cross at 90 deg. The loading is perpendicular to the plane of the bars, and consists of either a uniform load or of concentrated loads acting at the crossing points of the bars. The method of solution given is applicable to areas other than square, provided that an orthogonal system of bars is employed.

In the solution, the bending moments and deflections of the bars are first obtained under the assumption that crossing bars are not attached at their junction; then statically indeterminate forces are introduced at these points, and are determined from the condition that there can be no relative displacement of two bars at the points of crossing if, as is assumed, the bars are in reality attached. A numerical example which is worked out for a square area shows that this method is comparatively easy to apply.

The results show that the bending moment variation in the system of ribs is very similar to the corresponding variation in a square plate, while the rigidity of the ribs is, of course, lower than that of the plate. The conclusion is reached that this type of structure is particularly efficient from a weight standpoint when large areas are to be covered and bars of equal bending rigidity are used in both directions. Bruno A. Boley, USA

862. **Stanley U. Benscoter, "Shear flows in multicell sandwich sections,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1749, Nov. 1948, 28 pp.

A second-order linear difference equation is developed relating the shear flows of three successive cells of an n-celled constant-height closed structure, to the rate of twist of the second cell. The shear flow due to bending is not taken into account. The difference equation is solved for shear flows with constant rate of twist (pure torsion), linearly varying twist which corresponds to beam bending, sinusoidal twist, and arbitrary twist distribution. In the last case the twist is first expanded into a finite Fourier sine series, and then the shear flow in any cell is obtained as a linear function of the twist in a matrix form. Finally a formula is obtained for the torsion constant.

Several numerical examples are worked out. Formulas developed may conveniently be used for engineering computations. By coincidence an unpublished solution of a similar problem suggested by H. Reissner was obtained two years ago by this reviewer. A. Cemal Eringen, USA

863. **American Railway Committee on Impact, "Advance report of the Committee on Impact and Bridge Stresses,"** *Bull. Amer. Rly. engng. Ass.*, Sept.-Oct. 1948, vol. 50, pp. 53–146.

This report embodies the results of tests made on the effect of impact and secondary stresses on eight railroad bridges. It is found that such effects are generally small and well within design specifications. Ernest F. Masur, USA

864. **D. A. Stan and A. Tauber, "Designing of reinforced concrete column footings"** (in Rumanian), 1948. Paper, 8.2 × 5.7 in., 159 pp., 25 figs.

A method for designing square and rectangular reinforced-concrete column footings is presented, based on the following principles: The footing block is considered to be formed by four trapezoidal cantilever slabs, fixed in the column but independent of each other. Each of these slabs is considered to be formed of a central cantilever beam, having the width of the column, and by two built-in triangular slabs. Any connection between the adjacent slabs along the diagonals is completely neglected, an assumption which the authors state to be justified as it is on the safe side.

Using the German standards for designing reinforced-concrete constructions, the authors establish formulas for the theoretical height and for the areas of the reinforcing bars of the footings. The work is supplemented by 92 tables containing the necessary data for square and rectangular footings, for soil pressures varying from 7.11 to 56.9 psi, for concrete working stress from 355 to 640 psi and for a working stress for the steel of 17,068 psi.

The authors show by comparative tests that the design method proposed by them gives a higher safety factor than other systems in use. Aurel A. Beles, Rumania

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 839, 843, 845, 850, 871, 872, 874)

865. **C. C. Chow, A. W. Dana, and G. Sachs, "Stress and strain states in elliptical bulges,"** *J. Metals*, Jan. 1949, vol. 1, pp. 49–58.

Proposed general laws of plasticity, designed to define the stress-strain curves for all stress states, are usually proved by testing thin-walled tubes under combined hydrostatic pressure and uniaxial tension. However much larger strains than can be obtained in this manner may be obtained by subjecting thin membranes to a hydrostatic pressure, the membranes forming deep bulges. The authors analyze previous tests of Sachs, Espey, Kasik, and Lubahn [*Trans. Amer. Soc. mech. Engrs.*, 1946, vol. 68, p. 161 and

p. 271] on three bulge shapes, one circular and two elliptical, in which the ratio of the two positive principal strains varied between 1.0 and 0.35. The material used was cartridge brass with 70 per cent copper and 30 per cent zinc. The strains were measured along the lines of photogrids, initially located parallel to the axis of the clamping dies.

The analysis of the experimental results has shown that none of the known laws of plasticity agrees accurately with the stress-strain curves for the stress states of the bulges. In particular the stresses for biaxial tension with equal principal stresses, present in circular bulges, are considerably higher than for uniaxial tension. The authors assume that this may be due to the nonhomogeneity of the strain across the contour of a bulge.

Albert Kochendörfer, Germany

866. John E. Dorn, "Stress-strain rate relations for anisotropic plastic flow," *J. appl. Phys.*, Jan. 1949, vol. 20, pp. 15–20.

A stress-strain rate law for anisotropic plastic flow is developed on the two hypotheses that the plastic-strain rates are functions of the stresses, and the components of the strain rates are linear functions of the stresses. In general, there are 36 independent coefficients of plasticity; but this number is reduced when symmetries are present. These coefficients depend on the history of flow and deformation. Formulas for the coefficients in rotated coordinate systems are developed. It is also assumed that these coefficients remain constant over nominal deformations.

If the increase in stress is assumed to be a function of the plastic strain energy, the generalized flow stress σ and generalized strain φ can be computed in terms of the stresses, strain rates and coefficients of plasticity. Curves of σ versus φ are given for pure tension and equal biaxial tension; these agree well for small strains. Curves of yielding under combined stresses in anisotropic plastic flow are also given. G. H. Handelman, USA

867. G. W. Scott Blair and J. E. Caffyn, "An application of the theory of quasi-properties to the treatment of anomalous strain-stress relations," *Phil. Mag.*, Jan. 1949, vol. 40, pp. 80–94.

As the authors state, the idea of quasi properties, which was proposed in an earlier paper, is extended to cover anomalous strain-stress relations. Certain experiments on the relaxation of plastics at constant stress were selected on account of their exceptional divergence from the Nutting relaxation law. It was shown that for the less serious anomalies, an equation integrated from the strain-stress fractional differential in some cases fitted the data better than the corresponding strain-time equation. For extreme anomalies, an integrated form of the previously proposed double-fractional differential equation, reduced to four variables, is shown to fit the data admirably.

It is claimed that, when the aim is to relate the complex rheological behavior of materials to the results of tests in which their firmness is assessed subjectively by handling, it is better to express the data in terms of quasi properties rather than by the classical methods used to study the structure of materials.

Henri M. Schnadt, Luxemburg

868. R. P. Carreker, J. G. Leschen, and J. D. Lubahn, "Transient plastic deformation," *Metals Technol.*, Sept. 1948, vol. 15, T. P. no. 2477, pp. 1–8.

Creep experiments on lead wires showed that an increase in stress from 456 psi to 468 psi for a short time caused a considerable rise in the speed of deformation. Upon subsequent return to the original stress, the creep first came to a complete stop, and then gradually increased to its original speed. The time which elapsed before the creeping process started up again depended upon the length of time the increased load was applied. In another experiment, when the stress was alternately increased and decreased 2 per cent above and below a steady value of 456 psi, the elongation in 5 hr was 1.5 times the elongation of a similar specimen under a constant stress of 456 psi.

Creep experiments on lead wires in which the temperature was increased for a short time from 29.5 C to 54 C showed an increased creeping speed at the higher temperature and, after return to the original temperature, an incubation period of several hours during which no creep took place.

Very noticeable transition effects were observed when the speed of unit elongation was suddenly decreased from 8.9×10^{-3} per min to 3×10^{-4} per min. In this case, the tension first decreased considerably and then slowly increased again. On the other hand, a transition back to the original speed caused at first a rather large increase in tension. A particularly conspicuous minimum occurred in the stress-strain curve when the speed of unit elongation was decreased from 1.1×10^{-2} per min to 3×10^{-5} per min. Similar observations were made on copper specimens. Erich Siebel, Germany

Failure, Mechanics of Solid State

(See also Revs. 839, 867, 883)

869. R. Cazaud, "The fatigue of metals (La fatigue des métaux)," Dunod, Paris, 1948. Paper, 9.2 × 6 in., 318 pp., 241 figs.

This book gives a complete and very clear account of experimental and theoretical results on fatigue of metals. Since the last edition was published in 1943 much research work on the subject has been published, and this new edition illustrates the latest achievements in this important field.

After a rapid résumé of the history of fatigue tests and theories, from those first proposed by Albert and Wöhler in Germany and by Rankine and Hodgkinson in Great Britain up to the latest results, the characteristics of fatigue failure are discussed. Different methods for detecting fatigue fractures are explained, including electromagnetic, magnetoacoustic, X-ray analysis and ultrasonic methods. Various theories of fatigue failure are discussed in detail, including the latest failure theories proposed by Dehlinger and Orowan.

One entire chapter is devoted to fatigue tests and a comprehensive description is given of the different machines employed for such tests. Another chapter deals with the fatigue limits of steels and metallic alloys, including tables of data. Various factors influencing fatigue are then analyzed, including the influence of loading, speed, overstressing, understressing, rest, the size and shape of the specimens, surface conditions, temperature, and corrosion and other chemical actions. The problem of welding and the resistance of welded joints to fatigue failure are examined and discussed. The last chapter deals with the important problem of improving the resistance of machinery to fatigue failure.

A complete bibliography is given at the end of each chapter. The text is illustrated with many tables, graphs, and photograms. This book will be of great help to workers in a wide field of research. Enrico Volterra, USA

870. Günter Wassermann, "On the effect of stress and temperature on stress corrosion (Über die Spannungs- und Temperaturabhängigkeit der Spannungskorrosion)," *Z. Metallk.*, Mar. 1948, vol. 39, pp. 66–71.

Straight-line plots on a double logarithmic scale of stress against lifetime in stress-corrosion tests are known to fit experimental results fairly well, even for comparatively large variations

in lifetime [W. C. Schroeder, A. A. Berk, and E. P. Partridge, *Proc. Amer. Soc. Test. Mat.*, 1936, vol. 36, p. 721]. The author confirms this rule for a series of investigations in the literature (including some of his own) and finds that it can be extended to hold for similar plotting of lifetime against temperature at constant stress. The influence of cold work is discussed. Due to considerable scatter of the experimental values it is necessary to use a large number of specimens. Folke K. G. Odqvist, Sweden

Design Factors, Meaning of Material Tests

(See Revs. 869, 872, 873, 882)

Material Test Techniques

(See also Revs. 929, 955)

871. **W. E. Carrington and Marie L. V. Gayler, "The use of flat-ended projectiles for determining dynamic yield stress, III. Changes in microstructure caused by deformation under impact at high velocity,"** *Proc. roy. Soc. Lond. Ser. A*, Sept. 2, 1948, vol. 194, pp. 323–331, 8 plates.

Hardness traverses and metallographic examinations were made on impact specimens of steel, duralumin, and 92.5 per cent Ag—75 per cent Cu after impact on steel plates. Cracks, "compression bands" (possibly caused by movement of wedges of material within grains), and twins were noted prior to the occurrence of slip. Accelerated aging and possible local melting were also present because of the high temperature produced by impact. All metallographic features are well illustrated by plates.
A. R. Bobrowsky, USA

872. **R. L'Hermite, "Researches on the penetration of conical probes and the problem of hardness (Recherches sur la pénétration des pointes coniques et le problème de la dureté),"** *Bâtim. Trav. publics*, Nov. 1948, no. 50, pp. 1–4.

It is assumed that a cone or a ball penetrates into a plastic body if the mean normal pressure and the mean tangential traction at the surface of contact reach values characteristic of the material (the connection of the mean tangential traction with the coefficient of friction is not considered). The relationship between force and penetration obtained from this assumption is compared with experiments. E. Orowan, England

873. **P. W. Rowe and C. Gurney, "Tearing experiments on metal sheets,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2282, Feb. 1945 (issued in 1948), 6 pp.

The load required to tear a bent-up tab from a thin sheet has been measured for several metals. The apparatus and specimens are described in detail but it may be noted that the bend radius at the root of the tab and the saw cut between the sides of the tab and the adjacent material of the sheet are not specified. These details probably would have a significant effect on the tearing strength and should be standardized if tests of this type are to achieve wider use. The most significant conclusion to be drawn from the tests is that the tearing strength for a given material varies as the square of the thickness. John E. Goldberg, USA

874. **Raymond Heydel, "Internal friction of metals (Le frottement interne des métaux),"** *Bâtim. Trav. publics*, Nov. 1948, no. 48, pp. 1–22.

This reviews of internal friction, puts the emphasis on experimental methods for determining it. A brief account is given of the most important experimental observations, and of recent French hypotheses about the causes of internal friction.
E. Orowan, England

Mechanical Properties of Specific Materials

(See also Revs. 841, 867, 870, 873, 874, 929)

875. **Werner Köster, "The variation with temperature of the modulus of elasticity of pure metals (Die Temperaturabhängigkeit des Elastizitätsmoduls reiner Metalle),"** *Z. Metallk.*, Jan. 1948, vol. 39, pp. 1–9.

The variation with temperature of the modulus of elasticity has been examined for 32 pure metals, within a range of temperatures from −180 C to +1000 C. The moduli were determined from transverse vibration tests of straight prismatical bars.
Ragnar Nilson, Sweden

876. **Werner Köster, "An anomaly in the variation with temperature of the modulus of elasticity and the damping of copper, silver, aluminum, magnesium (Über eine Sondererscheinung im Temperaturgang von Elastizitätsmodul und Dämpfung der Metalle Kupfer, Silber, Aluminium und Magnesium),"** *Z. Metallk.*, Jan. 1948, vol. 39, pp. 9–12.

In connection with the investigation of the temperature variation of the modulus of elasticity of pure metals reviewed in the preceding review, the author has found that some of the metals do not behave normally. Within a wide range of temperature the modulus of elasticity becomes smaller and the internal damping capacity greater than expected. This anomalous behavior is examined and shown to be dependent on certain impurities of the metals. By treating the metals with deoxidizing substances the behavior becomes normal. The author expresses the opinion that the influence of oxygen on the diffusion of extraneous atoms is the cause of the abnormal behavior. Ragnar Nilson, Sweden

877. **G. Siebel, "The reduction of grain size in magnesium alloys for castings (Über die Kornverfeinerung von Magnesium-Gusslegierungen),"** *Metall*, Nov. 1948, no. 21/22, pp. 357–363.

The grain size of cast alloys can be refined in two ways, by superheating or by treating the metal with a material which contains carbon Both methods give good grain refinement and higher mechanical properties for alloys that have a rather high percentage of Al. For the Mg-Zn alloys the refinement effect is zero or negative. A good effect is also obtained when materials containing chlorine are used for the refinement process, and the effect seems to be the same whether inorganic or organic substances are used. In the case of the superheating method it is made clear that the purity of the crucible has a great influence on the results. The author has done extensive experimental research on the problem and all the results which are reported in the article indicate the same behavior. Ragnar Nilson, Sweden

878. **Hugo Vosskühler, "Contribution to the problem of defining the creep strength of light-metal alloys (Beitrag zur Frage der Definition der Dauerstandfestigkeit von Leichtmetall-Legierungen),"** *Z. Metallk.*, Mar. 1948, vol. 39, pp. 79–87.

After reviewing and criticizing the German literature on short-time creep tests of light-metal alloys, the author describes long-time creep tests extended over 1000 days at constant temperatures of 30 and 150 C for a number of magnesium and aluminum alloys, including cast and malleable alloys. The rupture stress is plotted against lifetime, the former on a linear and the

latter on a quadratic scale. The curves generally become horizontal for lifetimes of the order of a few hundred days. From a large number of short-time creep tests of the German standard type (25 to 35 hr, as recommended for steels) curves for total strain being plotted against time, it is concluded that the point of inflection is generally to be found at a creep velocity of about 5×10^{-4} per cent per hr. The best agreement between long-time tests and short-time tests for determining the limiting creep stress is obtained for a creep velocity of 10×10^{-4} per cent per hr. The author finds support for a recommendation that the short-time standard test be used also for light-metal alloys.

Folke K. G. Odqvist, Sweden

879. E. G. Chilton, "Large deformations of an elastic solid," *J. appl. Mech.*, Dec. 1948, vol. 15, pp. 362–368.

Hencky [*Trans. Amer. Soc. mech. Engrs.*, 1933, vol. 55, p. 55] has given equations which, under simplified conditions, describe the stress-strain relations for elastic deformations beyond the limits of Hooke's law in the form:

$$\sigma_x - \sigma = 2G[\phi_x e^{\phi_x} - (\phi_x e^{\phi_x} + \phi_y e^{\phi_y} + \phi_z e^{\phi_y})/3], \text{ etc.}$$

where σ_x, σ_y, σ_z are the principal stresses, σ is the hydrostatic stress defined by $\sigma = (\sigma_x + \sigma_y + \sigma_z)/3$ and ϕ_x, ϕ_y, ϕ_z are the distortional strains $\phi_x = \epsilon_x - \epsilon, \ldots$, which cause no change in volume.

The author applies these equations to some fundamental types of stressing: tension, compression, bending, shear and torsion. He verifies by experiments with vulcanized rubber that Hencky's theory describes satisfactorily the deformations of this material within the limits of practical applications.

Franz Wever, Germany

880. Takuzo Sakai and Akira Isihara, "On the statistical theory of rubber-like elasticity," *J. Colloid Sci.*, Feb. 1949, vol. 4, pp. 71–77.

The elastic properties of chainlike polymers are treated by classical statistical mechanics. The entropy of the system is obtained as a function of the external dimensions and of parameters describing the size and configuration of the chains, under conditions of thermodynamic equilibrium. From this result and a few simplifying assumptions it is shown that the modulus of elasticity E_0 for infinitesimal strains is given by

$$E_0 = 3\rho R T \lambda_0^2/m$$

where λ_1 is the original distance between the two ends of a chain divided by the length of the chain, and m is the molecular weight of an effective link of the chain.

The same result had been derived previously in a less general manner by T. Sakai [*Proc. phys. Soc. Japan*, 1943, vol. 17, p. 273] and by James and Guth [*J. chem. Phys.*, 1943, vol. 11, p. 455].

Serge Gratch, USA

881. G. M. Bartenev, B. A. Dogadkin, and N. M. Novikova, "Mechanical properties of vulcanized butadiene-styrene rubber under two-dimensional deformation" (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), Oct. 1948, vol. 18, pp. 1282–1289.

The theory of the elasticity of rubber, as discussed by Treloar and Wall, applies more readily to the case of two-dimensional deformation than to the case of linear deformation. This is due to the effect of "mechanical vulcanization," which reduces plasticity and crystallization of the material.

In the present paper an experimental study of two-dimensional deformation of synthetic rubber and its vulcanizers is described. Test specimens had been vulcanized for different lengths of time. Comparing the experimental stress-deformation curves with theoretical curves, it can be seen that correlation is closest in those cases in which the sulphur content did not exceed 1.5 per cent.

Andrew Brodsky, USA

882. J. R. Shank, "Plastic flow of concrete at high overload," *J. Amer. Concr. Inst.*, Feb. 1949, vol. 20, pp. 493–498.

The author re-examines a formula which he had previously given for the plastic flow of concrete, in the light of data from several sources. This formula, $y = Cx^{1/a}$, where y is the strain corresponding to unit stress, x the time in days, and C and a are supposed to be experimentally determined constants, does not appear to be correct. The data would indicate that C actually varies linearly with stress up to a stress of about 70 per cent of the ultimate, with a progressively more rapid increase thereafter. These data also indicate that the ultimate strength in long-time loading is in the neighborhood of 90 per cent of the strength determined in a standard static test. The author suggests a re-evaluation of safety factors on the basis of these data.

Morton B. Millenson, USA

Mechanics of Forming and Cutting

(See also Rev. 865)

883. R. Howard, "Contact loading," *Auto. Engr.*, Jan. 1949, vol. 39, pp. 10–12.

The calculation of maximum shear stress caused by contact between parallel cylinders (Hertz's theory) is reviewed. It is stated that the calculated depth of maximum shear agrees closely with observed depths of pitting, and it is therefore suggested that this depth should be taken as a factor in determining the thickness of a case-hardened layer.

P. S. Symonds, USA

Hydraulics; Cavitation; Transport

(See Revs. 889, 891, 919)

Incompressible Flow: Laminar; Viscous

(See also Revs. 838, 903, 904, 905, 918, 947, 948)

884. Toshimitsu Nagakura, "Generalization of the airfoil transformation function" (in Japanese), *Proc. phys. Soc. Japan*, Nov.-Dec. 1947, vol. 2, pp. 167–176.

The author proposes, as a generalization of the Joukovski transformation, a transformation function of the form

$$\frac{u-1}{u+1} = \left(\frac{t-1}{t+1}\right)^{k(t)} e^{h(t)}$$

by means of which an airfoil section in the u-plane is transformed into a circle in the t-plane. Suitable functional forms are suggested for $k(t)$ and $h(t)$ which allow the geometrical parameters of the airfoil to vary over a wide range. Some discussion of numerical calculations is given, but no example of the calculation of an airfoil with given parameters is worked out.

Itiro Tani, Japan

885. A. E. Green, "The two-dimensional aerofoil in a bounded stream," *Quart. J. Math.*, Sept. 1947, vol. 18, pp. 167–177.

The airfoil in a stream bounded by a plane wall is studied. The method used is conceptually simple, and follows a classical pattern. The complex potential is given as a sum of that of a uniform flow, a vortex and a series, plus the analogous terms due to reflection at the wall; then the Blasius formulas are applied

The unknown coefficients are determined by transforming the airfoil into a segment, requiring that it be a streamline, and imposing the Joukovski condition.

The calculations are, of course, somewhat complicated, and the method is applied only to an airfoil of a circular-arc profile, carrying the calculation to the first terms of the series only.

There follows some discussion of the influence of camber and thickness at varying distance of the airfoil from the wall. The method is susceptible of application to the study of an airfoil in a tunnel. Gino Moretti, Argentina

886. **Goitiro Miyasaka and Kōetu Simizu, "The calculation of an airfoil with given pressure distribution"** (in Japanese), *Trans. Soc. mech. Engrs. Japan*, 1947, vol. 14, no. 46, pp. 171–180.

An approximate formula has been given by T. Moriya [*J. Soc. aero. Sci. Japan*, 1937, vol. 5, p. 7], which permits calculating the pressure distribution around an airfoil of arbitrary shape. Using this formula, the authors propose a method of successive approximation for solving the inverse problem, that is determining the airfoil shape when the pressure distribution is given. Numerical examples are given in connection with three NACA airfoils.

Itiro Tani, Japan

887. **Goitiro Miyasaka, "Discontinuous potential theory of arbitrary wing sections"** (in Japanese), *Trans. Soc. mech. Engrs. Japan*, 1947, vol. 13, no. 45, pp. 206–217; vol. 14, no. 46, pp. 181–192.

The author extends Witoszyński's theory of single burbling [W. F. Durand, *Aerodynamic Theory*, vol. 3, pp. 1–33] to the airfoil of arbitrary section. The complex velocity potential for the flow past a circle is assumed in the form

$$-u(e^{i\alpha}z + a^2e^{-i\alpha}/z) + iK \log(z/a),$$

where the separate terms correspond to the translatory flow, the circulation around the circle, and the layer of discontinuity, respectively. It is then transformed conformally into the flow past an airfoil; the lift, profile drag, pitching moment and pressure distribution acting on the airfoil are determined in the usual way. An example is worked out on the airfoil NACA 4412, for which the calculated values are in good agreement with NACA experiments, except near the stall. Itiro Tani, Japan

888. **Kazuo Kondo, "On the practical solution of an integral equation with a singular kernel"** (in Japanese), *Appl. Math. Mech.* (*Ōyō Sūgaku Rikigaku*), Nov.-Dec. 1947, vol. 1, pp. 259–277.

The integral equation in question is the fundamental equation of the lifting line theory of wings:

$$h(\eta)\varphi(\eta) + \frac{1}{2\pi}\int_{-1}^{1} \frac{d\eta'}{\eta - \eta'}\frac{d\varphi}{d\eta'} = g(\eta).$$

Approximate means of solution have been proposed by several authors; in these the second term on the left-hand side is replaced by a linear combination of the values of φ at suitable representative points [I. Tani, *Rep. aero. Res. Inst. Tokyo*, 1934, no. 111; 1940, no. 197; K. Wada and K. Toyoda, *J. aero. Sci. Japan*, 1936, vol. 3, p. 268; H. Multhopp, *Luftfahrtforsch.*, 1938, vol. 15, p. 153]. The author improves the method of Multhopp in that the representative points are distributed more closely in the region where it is necessary. This is done by writing the second term in the form $(1/2\pi)\int_{-1}^{1} J(\xi, \xi')\{d\xi'/(\xi - \xi')\}d\varphi/d\xi'$, where $J(\xi, \xi') = (\xi - \xi')/(\eta - \eta')$ and ξ is a monotonous function of η subjected to the condition: $\xi = \pm 1$ at $\eta = \pm 1$, and by assuming a suitable functional form for ξ. Numerical examples are given for elliptic wing having various (continuous or discontinuous) forms of angle-of-attack distribution along the span.

Itiro Tani, Japan

889. **D. A. Efros, "Calculation of the hydrodynamic forces acting on a cavitation contour in plane flow"** (in Russian), *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), Apr. 1948, vol. 60, pp. 29–31.

In an earlier paper [*Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), 1946, vol. 51, pp. 267–270] the author has considered a particular mathematical model, sometimes known as the re-entrant-jet model, for an attached cavitation bubble behind an obstruction in the form of an arc in a two-dimensional flow. This model was apparently first introduced by H. Wagner and has been studied by G. Kreisel, D. Gilbarg and D. H. Rock, M. I. Gurevich, and others. The re-entrant jet, which has been observed in experiment and which, if it does not first degenerate in some manner, will strike the back of the obstruction, is in the mathematical model conveniently removed onto a second sheet of a Riemann surface. In the author's earlier paper he studied the conformal-mapping problem associated with this model and, in order to define the solution uniquely, assumed that there was no circulation along a contour surrounding obstruction and bubble. In the present paper he allows circulation, assumes the lift is given entirely by the circulation according to the Kutta-Joukovsky formula, and is again able to define the solution uniquely, so that both lift and drag may be determined.

Courtesy of Mathematical Reviews J. V. Wehausen, USA

890. **O. H. Faxén, "Forces exerted on a rigid cylinder in a viscous fluid between two parallel fixed planes"** (in English), *Acta Polyt.*, 1947, no. 2, pp. 1–13.

The forces on a rigid cylinder in translatory motion relative to a viscous fluid between two parallel planes have been calculated for two cases: one in which the cylinder moves parallel to the boundaries through stationary fluid, and the other in which the cylinder is fixed relative to the plane boundaries and the fluid moves. These problems are solved under flow restrictions which justify neglecting the acceleration and velocity gradient terms in the classical equations of motion for an incompressible viscous fluid. A complete analysis is presented for only one of the cases.

In either case it turns out that the drag is approximately proportional to the ratio of cylinder diameter to the distance between the plane boundaries for values of this ratio between 0.05 and 0.3. The increase in drag is parabolic for values of this ratio above 0.3. The same relation holds for the pressure difference for a pair of points in the undisturbed fluid up- and downstream respectively from the cylinder.

The treatment of even the simplified problem required a considerable amount of algebraic and numerical manipulation, which the author handles in a masterly fashion. The reviewer feels that the results of the paper should be extended to indicate more specifically the effect of the simplifications made on the range of validity of the calculated results. M. G. Scherberg, USA

891. **Y. I. Frenkel, "On the behavior of liquid drops on the surface of a solid body. I. A rolling drop on a sloping surface"** (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), July 1948, vol. 18, pp. 659–667.

The author considers the problem of a two-dimensional liquid drop on a sloping surface, taking account of gravitational and capillary forces. He finds that if α^* is the critical angle of slope of

the surface beyond which the drop begins to roll, the following relation is satisfied:

$$2\sigma = gm \sin \alpha^*,$$

where σ is the surface tension of the gas-liquid interface, m is the mass (per unit length) of the drop, and g is the gravitational acceleration. The author was not able to find analytically the angles of contact of the drop at its forward and rear boundaries and, on the basis of experimental evidence, assumes them equal to $\theta_0 \pm \Delta\theta$, where θ_0 is the angle of contact for the drop on a horizontal surface. With this assumption he is able to find the shape of the drop. The author also discusses in a general way some dynamic and hysteresis problems associated with drops and bubbles, which he intends to study later.

Courtesy of Mathematical Reviews J. V. Wehausen, USA

Compressible Flow, Gas Dynamics

(See also Revs. 920, 923)

892. Brown Univ., Grad. Div. Appl. Math., "Summaries of foreign and domestic reports on compressible flow, vol. 5," *Hdqtrs. Air Mat. Comm. Dayton tech. Rep.*, no. F-TR-1168E-ND, Feb. 1948, 118 pp.

The previous four volumes have been described in Revs. **502, 503**, Mar. 1948 and Rev. **998**, June 1948. The present volume contains 25 summaries (by Brown University staff members) of eight German, two Italian, two Russian and thirteen English papers. D. R. Mazkevich, USA

893. L. Gabeaud, "Moisson number (Le nombre de Moisson)," *Mémor. Artill. fr.*, 1947, vol. 21, no. 4, pp. 857–868.

The author reviews the early history of the idea of the Mach number. He advocates the priority of the French general, Moisson (1842–?), who anticipated the sonic barrier of Mayevski (1870) in 1868, the photographic methods of Mach (1887) in 1873, and the explicit use of "V/C" of Sarrau (1883) in 1883, with the support of wave-propagation arguments. Harvey Cohn, USA

894. Max M. Munk and Robert C. Prim, "On the canonical form of the equations of steady motion of a perfect gas," *J. appl. Phys.*, Oct. 1948, vol. 19, pp. 957–958.

By using the reduced velocity of the fluid, which is the ratio of the fluid velocity to the maximum velocity along any streamline, the authors decrease the number of basic equations for the steady motion of a perfect gas from five to four. These the authors call the canonical form. This result was previously obtained by B. Hicks, P. Guenther, and R. Wassermann [*Quart. appl. Math.*, 1947, vol. 5, pp. 357–361].

Courtesy of Mathematical Reviews H. S. Tsien, USA

895. J. Fabri, "A rapid method for determining characteristics of high-speed gas flow (Méthode rapide de détermination des caractéristiques d'un écoulement gazeux à grande vitesse)," *Off. nat. Étud. Rech. aéro. Rep.*, no. 17, 1948, pp. 1–39.

The paper gives a graphical method for computing a steady one-dimensional flow of a gas in a duct with variable cross section, taking into account the friction at the walls. The pressure drop due to the friction is assumed to be proportional to the stagnation pressure, to a function of the Reynolds number, and inversely proportional to the hydraulic radius. First the general equations for such a flow are given. Then they are integrated for a cylindrical tube, and for other cases in which the cross section varies in such a way that the integration can be carried out in a closed form. The results are represented in graphs. More general flows are subdivided into portions in which the closed solutions can be used as an approximation. Numerous examples show the application of the method. Gottfried Guderley, USA

896. Yosio Akita, "On the nonlinearity of gas dynamics" (in Japanese), *J. Soc. appl. Mech. Japan*, Mar. 1948, vol. 1, pp. 48–56.

Based on various methods of approximation, the conjecture is put forward that the occurrence of shock waves in the flow past a given obstacle is due to $\partial\varphi/\partial M$ becoming infinite, where φ is the velocity potential and M is the Mach number.

First, following Gröbner [*Luftfahrtforsch.*, 1943, vol. 20, p. 184] the velocity potential is assumed to be expressible in the form:

$$\varphi = r \cos\theta + \Sigma A_m(\theta)\, r^{-m},$$

(r, θ) being polar coordinates A_1, A_2, A_3 are explicitly determined, each containing two arbitrary constants. On account of the boundary conditions, these constants are determined by a system of quadratic equations involving M as a parameter. For flow past a circular cylinder it is found that $\partial\varphi/\partial M = \infty$ for $M = 0.388$, which the author considers to be the critical Mach number corresponding to the breakdown of continuous irrotational flow.

Secondly, Galerkin's method is applied to the flow past a circular cylinder, assuming

$$\varphi = (r + r^{-1})\cos\theta - a_1(r^{-1} - r^{-3}/3)\cos\theta - a_2(r^{-3}/3 - r^{-5}/5)\cos 2\theta.$$

It is found that for a symmetric flow ($a_2 = 0$) $\partial a_1/\partial M = \infty$ for $M = 0.653$, and that asymmetric flow can take place from $M = 0.653$ onward.

Thirdly, a numerical method of calculation for flow past arbitrary thin airfoils is proposed, on the basis of the variational principle as considered by Braun [*Ann. Phys.*, 1932, vol. 15, p. 645]. Two examples are given, the NACA 0012 and NACA 4412 airfoils, both at zero angle of incidence. It is remarkable that the minimum pressure point moves rearward as the Mach number is increased, and the pressure coefficient increases more rapidly with Mach number than is expected from the Prandtl-Glauert relation. However, it is to be noted that the lift coefficient of NACA 4412 increases more slowly than the prediction of the Prandtl-Glauert relation, and, moreover, it attains the maximum at a certain Mach number below the critical Mach number and then decreases rapidly. Isao Imai, Japan

897. Hirosi Hatanaka, "On the stability of a surface of discontinuity in a compressible fluid" (in Japanese), *J. Soc. Sci. Culture Japan*, Apr. 1947, vol. 2, pp. 3–7.

The stability of a surface of discontinuity between two parallel gas streams with different velocities, sound velocities and densities is considered using the method of small perturbations. It is concluded that instability is reduced as the Mach number is increased, but for stability at least one of the streams is required to have a supersonic velocity relative to the discontinuity surface. Isao Imai, Japan

898. J. C. Gunn, "Linearized supersonic aerofoil theory. Parts I and II," *Phil. Trans. roy. Soc. Lond. Ser. A*, no. 820, Dec. 9, 1947, vol. 240, pp. 327–373.

The development is based on the Laplace transform (with respect to the velocity direction angle) applied to the linearized potential equation. This simpler "subsidiary equation" is then solved by different expedients depending on the problem, making

use of the Laplace transform of the boundary conditions on the upwash. A Fourier-integral representation is used for the case of zero incidence. For lifting wings a very general Green's function method, analogous to that employed in diffraction problems, is developed. Raked-out wing tips are handled by a rotation of axes. The solution is completed by obtaining the inverse transform.

The method is applied to give the pressure distributions in closed form for a number of the simpler (from the vantage point of 1949) wings at zero and nonzero incidence. Included are the finite rectangular wing, both flat and with sinusoidal twist (washout), wings with curved supersonic leading edge and streamwise tips, trapezoidal wings, and apex-forward triangular wings, unyawed and yawed, with at least one leading edge supersonic. The triangular wing with both leading edges subsonic is treated as the limiting case of a trapezoid, by an iteration method based on cancellation of lift, leading to an infinite series.

The solution for the lifting rectangular flat plate of low aspect ratio is extended back along the chord behind the conical-flow region by superposing solutions according to the principle of cancellation of lift. Numerical results are obtained for

$$A(M^2 - 1)^{1/2} = \tfrac{1}{2}.$$

The treatment is less complete and less elegant than that of Lagerstrom.

The interest in this paper centers in its contributions to applied mathematics. All of the problems considered have been solved much more simply or, in the case of the triangle within the Mach cone, more exactly, by other means, in particular in the work of Evvard, Lagerstrom, C. E. Brown, and, very recently, Goodman of Cornell Aeronautical Laboratory. H. S. Ribner, USA

899. **J. Nicolas and H. Audic, "Determination of properties of a perfect fluid for various shock-wave combinations in two-dimensional flow (Détermination des caractéristiques d'un fluide parfait pour diverses combinaisons d'onde de choc en écoulement plan),"** *Rech. aéro. Paris*, Jan.-Feb. 1949, no. 7, pp. 51–58.

In air intakes at supersonic velocities it is desirable that the normal shock wave be preceded by one or more oblique ones. From the known equations of the shock wave, and assuming as parameters the Mach number of the flow before the air intake and the inclinations of the walls at the inlet, the pressure ratio, the impact-pressure ratio and the thermic efficiency for 0, 1, 2 oblique shock waves are calculated. The results are presented in graphs, giving the wall inclinations of maximal efficiency; these maxima are not critical. It appears that one oblique wave for M between 1.6 and 2.1 and two for M between 2.1 and 2.6 are necessary. Experimental data are wanting.

Gino Moretti, Argentina

900. **A. R. Manwell, "The analysis of subsonic flow and constant velocity aerofoils,"** *Phil. Mag.*, Sept. 1948, vol. 39, pp. 712–722.

The author points out that by a change of scale the differential equations for compressible flow in the hodograph plane can be considered as those of incompressible flow in a shallow dish with the depth of fluid specified as a function of the distance from the center of the dish. In the case of the von Kármán-Tsien approximation, this depth of fluid is a constant and thus the method of complex variables can be applied. The author further suggests that the boundary of the profile be specified as a line in the hodograph plane so as to simplify the mathematical problem. He then uses this approach to obtain the solution in closed form for a family of symmetric profiles at zero angle of attack with almost constant velocity over the surface. For small thickness ratios, his profile is very similar to an ellipse of the same thickness ratio.

Courtesy of Mathematical Reviews H. S. Tsien, USA

901. **E. A. Karpovich and F. I. Frankl, "Resistance of a delta wing in a supersonic flow"** (in Russian), *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, July-Aug. 1947, vol. 11, pp. 495–496.

A direct integral expression for the leading-edge suction of a flat lifting delta wing in a supersonic stream is set up by applying the momentum theorem to the fluid crossing two conical surfaces which enclose the two leading edges and converge upon them in the limit. Properties of linearized conical flows reduce the problem to evaluation of residues at the traces of the leading edges in the "representative projection plane." The results can be reconciled with those of Puckett and Stewart [*J. aero. Sci.*, Oct. 1947] and Brown [corrections to *Nat. adv. Comm. Aero. tech. Note*, no. 1183, Dec. 1946], obtained by less rigorous means.

M. V. Morkovin, USA

902. **Robert T. Jones, "Properties of low-aspect-ratio pointed wings at speeds below and above the speed of sound,"** *Nat. adv. Comm. Aero. Rep.*, no. 835, 1946 (printed in 1949), 5 pp.

For low-aspect-ratio pointed wings at small angles of attack the flow in each plane, perpendicular to the long (fore-and-aft) axis, is assumed to be two-dimensional. The lift appears to depend on the increase in spanwise width of the section up to the section of maximum width. Sections behind this one are assumed to contribute nothing to the lift (Kutta condition). The load distribution derived is elliptical and moreover independent of compressibility effects. It is not clear that the experiment described proves this last result. R. Timman, Holland

Turbulence, Boundary Layer, etc.

(See also Revs. 923, 951)

903. **Busuke Hudimoto, "On the fluid frictional resistance of a rough surface"** (in Japanese), *J. Soc. mech. Engrs. Japan*, Feb. 1947, vol. 50, pp. 55–56.

The author integrates the integral momentum relation of the turbulent boundary layer along a rough surface, assuming the velocity distribution to be the same as that measured by Nikuradse on roughened pipes. Numerical discussions are given only for the case of zero pressure gradient, for which an analysis has already been made by Prandtl and Schlichting. The author's calculation is somewhat crude but simpler than the previous ones; nevertheless the result it gives seems to be satisfactory for most practical purposes. Itiro Tani, Japan

904. **Busuke Hudimoto and Yūichi Iwagaki, "On the fluid frictional resistance of a rough rotating disk"** (in Japanese), *Trans. Soc. mech. Engrs. Japan*, 1947, vol. 13, no. 44, pp. 180–184; 1948, vol. 14, no. 47, part 1, pp. 44–49.

The authors calculate the frictional resistance of a rotating disk with a rough surface, under the assumption that the velocity distribution is the same as that measured by Nikuradse on roughened pipes. The second report refers to the case when the disk is rotating in a casing. Itiro Tani, Japan

905. **A. M. Feinzilber, "On a problem of chemical dynamics"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Oct. 1948, vol. 62, pp. 457–460.

The viscosity of an incompressible colloidal solution is taken to depend in a specific manner on the concentration. By suitable changes of variable the boundary-layer-type flow of such a solution can be reduced to the conventional boundary-layer-flow problem associated with a fluid of constant viscosity. The pressure gradient is taken to vanish in the analysis.
Courtesy of Mathematical Reviews G. F. Carrier, USA

906. H. Schuh, "On the solution of the equation of the laminar boundary layer along a plane wall for velocity and temperature fields with variable properties of state and for a diffusion field at high concentrations (Über die Lösung der laminaren Grenzschichtgleichung an der ebenen Platte für Geschwindigkeits und Temperaturfeld bei veränderlichen Stoffwerten und für das Diffusionsfeld bei höheren Konzentrationen)," *Z. angew. Math. Mech.*, May 1947, vol. 27, pp. 54–60.

The solution of the laminar boundary-layer equations along a heated or cooled flat plate is considered. The variations with temperature throughout the boundary layer of the density, viscosity and thermal conductivity are included, but the dissipation term in the energy equation is neglected. The solution, which involves a rapidly convergent step-by-step iterative process, is based on the method used by Piercy and Preston [*Phil. Mag.*, 1936, vol. 21, p. 995].

The paper describes the results obtained from numerical calculations in the following cases: (*a*) the flow of lubricating oil (Prandtl number 12.5 and 100) along a heated and cooled flat plate; (*b*) the flow of air (Prandtl number 0.7) along a heated flat plate.

The calculations show that, with air, the wall shear stress and heat transfer obtained by neglecting the variation of the physical properties of the fluid across the boundary layer ("isothermal" case) are within 5 per cent of the exact values, provided that the temperature difference is not too large. However, with lubricating oil the velocity and temperature distributions are very different from those obtained in the isothermal case.

The author further extends the method to the full solution of the compressible-flow boundary-layer equations in which the dissipation term is included, but no calculations are given. This solution is an alternative to that given by Crocco [*R. C. Circ. Mat. Palermo*, 1940/41, vol. 63].

Finally, the same method is applied to the problem of diffusion between two gases, taking into account the finite transverse velocity at the wall which is present with strong concentrations.
G. M. Lilley, England

907. B. Thwaites, "An exact solution of the boundary-layer equations under particular conditions of porous surface suction," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2241, May 1946 (issued in 1947), pp. 1–6.

An exact solution of the boundary-layer equations of motion is given for one case in which there is a continuous velocity at the boundary, normal to the boundary. The case is taken in which this suction velocity is proportional to the square root of the distance along the plate and there is a constant velocity outside the boundary layer. R. C. Binder, USA

908. B. Thwaites, "On certain types of boundary-layer flow with continuous surface suction," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2243, July 1946 (issued in 1947), pp. 1–6.

This report shows that the boundary-layer-velocity profiles tend to the Blasius profile at the front end of the plate in a uniform stream under constant continuous surface suction. The report also discusses the solution of the boundary-layer equations of motion when "similar" velocity profiles are assumed. It is shown that only two types of outside stream velocity distributions lead to similar profiles under ordinary conditions. R. C. Binder, USA

909. Goitiro Miyasaka and Kōetu Simizu, "The separation limit of the laminar and turbulent boundary layers" (in Japanese), *J. Soc. mech. Engrs. Japan*, Apr.-May 1947, vol. 50, pp. 127–129.

Prandtl has estimated the order of the pressure gradient against which a laminar boundary layer can flow without separation [W. F. Durand, *Aerodynamic Theory*, vol. 3, pp. 112–115]. The authors use the same method for a turbulent boundary layer, where the value of Buri's parameter is assumed to remain at the value corresponding to separation. It is demonstrated that the allowable pressure gradient is very large as compared with the case of laminar flow. Itiro Tani, Japan

910. H. C. Garner, "The development of turbulent boundary layers," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2133, June 1944 (issued in 1947), 21 pp.

The author gives a new method of calculating the development of a turbulent boundary layer. This method is a combination of the methods of von Doenhoff and Tetervin [see Rev. **991**, June 1948] and Howarth [see Goldstein, *Modern Developments in Fluid Dynamics*, vol. 2, pp. 374 and 436]. The new method utilizes the following equations: (1) The momentum equation in Howarth's form, with the exponent $^1/_4$ of R_θ replaced by $^1/_6$; the following equation is thus obtained:

$$\frac{d\Theta}{ds} = \frac{7}{6}\left[3 - \Gamma\left(H + \frac{13}{7}\right)\right]. \tag{1}$$

Here $H = \delta_1/\theta$, δ_1 is the displacement thickness, θ the momentum thickness, $\Theta = \theta R_\theta^{1/6}$, R_θ is the Reynolds number based on θ, $\zeta = (\rho U_1^2)^{-1}\tau_0 R_\theta^{1/6}$, τ_0 is the skin friction per unit length, U_1 the velocity at the edge of the boundary layer, $\Gamma = (\Theta/U_1)(dU_1/ds)$. (2) The formula of Falkner for skin friction which gives

$$\tau_0/(\rho U_1^2) = 0.006534\, R_\theta^{-1/6}.$$

(3) The following empirical equation for dH/ds, which is a simplified form of a similar equation used by von Doenhoff and Tetervin:

$$\Theta dH/ds = [\exp 5(H - 1.4)]\,[-\Gamma - 0.0135(H - 1.4)]. \tag{2}$$

The differential equations (1) and (2) are integrated for the NACA airfoils studied by von Doenhoff and Tetervin by the numerical method of Adams. Less labor is involved than in that method. The calculations indicate the rapid forward movement of the position of turbulent separation on thick airfoils with increase in C_L. They show that, while there is probably no turbulent separation on an 18 per cent low-drag airfoil at $C_L = 0$, the flow does separate at $C_L = 0$ near $s/c = 0.94$ (c is the chord of the airfoil) on a 24 per cent section. Ratip Berker, Turkey

911. A. A. Townsend, "Local isotropy in the turbulent wake of a cylinder," *Austral. J. sci. Res. Ser. A*, June 1948, vol. 1, pp. 161–174.

The author describes experiments made by him to investigate the validity of Kolmogoroff's theory of local isotropy for shear flows. In the wake of a circular cylinder, and in three traverses across the wake, he measured the mean squares of the spatial derivatives in the mean-stream direction of the three components of the turbulent fluctuation, for which the theory leads to the relation $\overline{U_x U_x} = \frac{1}{2}\overline{V_x V_x} = \frac{1}{2}\overline{W_x W_x}$. He also determined the

"skewness" and "flattening" factors (which should be constant in the local isotropy).

The results of the experiments confirm the theoretical predictions as far as the relative magnitudes of mean squares of velocity derivatives are concerned, although those concerning the constancy of the shape factors, such as $T_0(u)$, are confirmed for the central part of the wake only. The author found, however, that the flow is continuously turbulent in that part only, while it is only intermittently turbulent in the other parts. The introduction of an adequate intermittency factor to define the mean fractional duration of turbulent flow at any given point permits reconciling the experimental results with the theoretical ones and those obtained in previous measurements by the same author in flows possessing ordinary isotropy.

The results obtained encourage extending the theory of local isotropy to the study of shear flow. Carlo Ferrari, Italy

Aerodynamics of Flight; Wind Forces

(See also Revs. 887, 898, 902, 917, 956)

912. **W. J. Duncan and W. J. Scull, "Free motion of a stable glider in an atmosphere of variable density,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2081, Mar. 1941 (issued in 1948), pp. 1–13.

A method for the calculation of the glide path of a stable glider in an atmosphere of variable density is explained. The angle of incidence and the aerodynamic coefficients are assumed to be constant (no influence of Mach number). The equations of motion are solved by a step-by-step method, using the analytical solution in each interval. A numerical example for a glide in standard atmosphere, showing the percentage of overshoot as compared to the case of a homogeneous atmosphere, is included.

L. R. Lucassen, Holland

913. **S. Neumark, "The disturbed longitudinal motion of an uncontrolled aircraft and of an aircraft with automatic control,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2078, Jan. 1943 (issued in 1947), pp. 1–54.

This report presents an exhaustive study of the factors affecting the uncontrolled phugoid motion of airplanes. Practical approximate formulas for the roots of the "stability equation" are developed. These include terms often neglected, such as slip-stream effect and flight-path angle.

The theory is extended to include the action of the more common types of automatic control. Pickups sensing deviations in pitch angle, air speed, or angle of attack, and their derivatives and integrals, are considered to move the elevator through a servomechanism, producing pitching moments with no lag. The analysis is limited to the "stick-fixed" case of the uncontrolled motion, and to the "ideal-autopilot" case of the controlled motion. This should not appreciably detract from the generality of the conclusions concerning the long-period oscillation.

For the uncontrolled motion, the phugoid damping is shown to have a minimum with moderate static stability. The dynamic stability can be improved by increasing the airplane's rotary damping. Among the automatic controls, those sensing pitch angle, air speed or both are shown to be the most efficient. Manometric control (sensing air speed), however, subjects the airplane to a violent initial swing on encountering a longitudinal gust.

It may be true that phugoid stability bears little relation to handling qualities, but instability must always be suspect; and studies of the kind and amount of stability necessary for stable flight should interest aerodynamicist and autopilot designer alike. Dunstan Graham, USA

914. **Harry Greenberg and Leonard Sternfield, "A theoretical investigation of the lateral oscillations of an airplane with free rudder with special reference to the effect of friction,"** *Nat. adv. Comm. Aero. Rep.*, no. 762, 1943 (publ. in 1947), pp. 1–16.

The influence of various airplane parameters on the rudder-free short-period stability of typical aircraft configurations is studied. The motions taken into account are yawing of the craft about its center of gravity and rudder rotation. Interest is centered on control systems which have a stabilizing floating tendency, that is, which tend to oppose any disturbance of the craft.

For certain configurations it is found that the system is unstable for values of viscous control friction between two limits. These findings can in turn be used to predict in an approximate fashion the behavior of the system in the presence of solid friction in the control system. The solid friction is interpreted as an equivalent viscous friction dissipating the same energy per cycle of oscillation. The occurrence of steady oscillations under certain circumstances can then be inferred.

On the basis of the linearization of the problem, the effects of solid friction are thoroughly explored. The results of several numerical solutions for the motion of the nonlinear system in the presence of solid friction are compared with the approximate results and reasonable agreement is found.

Martin Goland, USA

915. **C. M. Britland, "Helicopter hovering performance,"** *J. roy. aero. Soc.*, Jan. 1949, vol. 53, pp. 67–76.

The purpose of this paper is to indicate in simple quantitative form the primary aerodynamic factors governing the performance of a hovering helicopter. The standard power relationships for hovering flight are interpreted in graphical form, and the effect of forward flight in limiting the choice of aerodynamic and operational parameters, due to the onset of blade stalling and compressibility, is illustrated by means of boundaries on these graphs. The power requirements in forward flight are not considered. Complete notations are introduced. The author states that the ratio (rotor horsepower)/(brake horsepower) is of the order of 0.85 for the "Sikorsky configuration" in vertical flight, and of the order of 0.90 for a twin-rotor helicopter.

The following three equations are given as the basis of the calculations:

Induced power: $P_i/W = (W/2\rho S)^{1/2}$; W is weight of helicopter, S area of disk, ρ air density.

Blade profile power:

$$P_p/W = \tfrac{1}{8}\rho C_d\sigma\ (S/W)\Omega^3R^3[1 + 3(\mu^2 + \lambda^2)];$$

σ is rotor solidity (blade to disk area), Ω angular velocity, R radius of rotor disk, μ disk flow ratio, λ tip speed ratio.

Rotor power:

$$P_r/W = P_i/W + P_p/W;\ RHP/W = (P_i/W + P_p/W)/550.$$

On this basis calculations and curves of W/RHP for varying solidities and tip speed are plotted against disk loading with $C_D = 0.010$ as the optimum profile drag coefficient. Stalling boundaries are found by making 16 deg the stall angle. Compressibility and reverse flow boundaries are also studied. The author summarizes the broad principles of hovering performance as follows:

(1) While reduction of rotor solidity and blade drag are both beneficial to power economy, it is unlikely that present-day performance as expressed by "pounds weight lifted per horsepower supplied to the rotor" can be improved upon by more than about 30 per cent, using these means alone.

(2) While the highest values of total lifting force per RHP are

generated by rotors operating at low disk loadings and low tip speeds, the effect of rotor blade weight is such that the greatest net lifting force per *RHP* will be generated at medium values of disk loading and medium or high values of tip speed, if the coning angle is limited to about 5 deg.

(3) If a high forward speed is required, the choice of design combinations of disk loading and tip speed is considerably limited by considerations of blade stalling and compressibility. There are consequent limitations on the lifting efficiency obtainable in hovering flight.

(4) For similar values of disk loading, and hence of rotor size, increase of hovering efficiency by decrease of solidity entails a lowering of the forward speed at which blade stalling will be encountered. Alexander Klemin, USA

916. John P. Reeder and F. B. Gustafson, "On the flying qualities of helicopters," *Nat. adv. Comm. Aero. tech. Note*, no. 1799, Jan. 1949, pp. 1–47.

Several problems pertaining to the flying qualities of current helicopters were investigated during flight and are presented in this paper.

In the author's opinion the problem of the instability due to the angle of attack of the rotor and the fuselage can be alleviated by means of a tail surface. By increasing the rotor damping without changing lag or providing an appropriate stick-force gradient, the control difficulties caused by high sensitivity in hovering can be reduced. Furthermore, undesirable transient control forces in maneuvers and excessive vibratory stick force should be prevented from reaching the pilot by means of irreversible mechanisms.
S. W. Yuan, USA

Aeroelasticity (Flutter, Divergence, etc.)

917. A. H. Hall, "Estimation of wing torsional divergence speed with downwash correction," *Nat. Res. Counc. Canada Aero. Note*, no. 2, 1948, 9 pp.

Torsional divergence speed is expressed in terms of two factors: one corresponding to the mode of elastic deformation, and the other correcting for the effect of downwash. The analysis is expressed in terms of a reference station by the semirigid theory widely used by the British. The two factors are computed for plain wings of aspect ratio 6 and plotted as a function of the taper ratio. The calculations are based on the elastic deformation corresponding to the first resonant mode, and the modes of induced angle of attack obtained from linear twist.
W. T. Thomson, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 847, 916)

918. Kazuo Kondo, "Calculation of a propeller by means of an asymptotic equation" (in Japanese), *Appl. Math. Mech. (Ōyō Sūgaku Rikigaku)*, Aug. 1947, vol. 1, pp. 97–108.

The author recapitulates his previous studies on propeller theory, with a view to presenting the most convenient method of calculating the aerodynamic characteristics of a given propeller. First, by using the asymptotic expansion, the induced velocity is expressed in a form similar to that for a monoplane wing. Second, the integration is performed by the method of representative points, the induced velocity being expressed as a linear combination of the values of the circulation at certain representative points. The integral equation for the circulation is then solved by the method of successive approximation. Two examples are given for testing the accuracy of the method.
Itiro Tani, Japan

919. J. Osterwalder, "The importance of cavitation in the development of high-pressure Kaplan turbine blading" (in English), *Escher Wyss News*, 1946/47, vol. 20, pp. 17–24.

Following the trend to extend the application of the Kaplan turbines towards higher heads, a great number of tests were carried out in the laboratories of Escher Wyss to develop cavitation-free blading for high-pressure turbines. Different methods to extend the range of cavitation-free operation are described and their respective advantages discussed. The latest development tends to blades with small changes in incidence from hub to tip, long blade sections at the hub (that is, small lift coefficients) and short sections at the tip, and specially developed airfoil-type sections with favorable pressure distribution.

The testing methods used (air test-rig, cavitation test-rig with stroboscope, etc.) are briefly described. E. Haenni, USA

920. Chung-Hua Wu and Lincoln Wolfenstein, "Application of radial-equilibrium condition to axial-flow compressor and turbine design," *Nat. adv. Comm. Aero. tech. Note*, no. 1795, Jan. 1949, pp. 1–101.

General equations governing three-dimensional flow through axial-flow machines are presented in order to determine the radial motion, velocities and pressure variation between blade rows. The radial pressure gradient between the blade rows is calculated using not only the simplified gradient due to the swirl between the rows, but the pressure gradient due to the radial motion as well. A sample calculation of a compressor and a turbine shows that inclusion of the radial motion has a considerable effect.

A typical example of a symmetric compressor of aspect ratio 2 shows that due to neglect of the radial motion the following errors were introduced: (1) Up to three degrees in flow angles, (2) nine per cent in relative Mach number, (3) only two per cent static-pressure rise across the rotor, as opposed to 13 per cent by the simplified method, (4) four per cent difference in mass flow, and (5) five per cent difference in permissible rotor speed.

The paper discusses also the compatible number of degrees of freedom available to the designer in specifying the radial variation of gas properties between blade rows. It is shown that under conventional design procedures the designer is free to specify two conditions at one axial station and one condition at each of the remaining stations. C. A. Meyer, USA

921. A. Pfenninger, "Further development of turbo-pumps" (in English), *Escher Wyss News*, 1946/47, vol. 20, pp. 25–32.

This paper gives a short survey of some new detail improvements in different varieties of turbopumps. For one-stage radial pumps of medium specific speeds a new volute construction was developed and tested. This dispenses with guide vanes at the inlet of the volute. Nearly the same peak efficiencies were reached with the new design, with flatter capacity curves.

A new design of mixed-flow pump with axial discharge has been developed, which can easily be built into the piping. A new two-vane impeller for drainage pumps gives maximum stage efficiencies of 84 per cent and is not subject to choking even with a high percentage of foreign matter in the delivered fluid.

Special stuffing boxes and shut-down seals for acid pumps are described. A number of tests for high-pressure-pump labyrinth-seals were carried out on scaled-up models which were tested with air at the Reynolds numbers corresponding to the operating conditions with water. E. Haenni, USA

Flow and Flight Test Techniques

(See also Rev. 885)

922. **Richard J. Moberg and Edward B. Palazzo, "The NACA hydraulic torque system for indicating propeller torque,"** *Instruments*, Dec. 1948, vol. 21, pp. 1100–1102.

A new torque dynamometer for wind-tunnel testing of propellers incorporates a hydraulic transmitter. The propeller-drive motor is mounted on ball bearings and restrained from rotation by two master cells. The slave cells connected to a Toledo printweight scale have their displacements indicated by electrical contacts. Displacement of the master cells under the propeller torque closes one contact, and the scale mechanism then operates to return the cell piston and therefore the master cell to its original position. Calibrations showed a maximum error of three per cent, with the maximum error less than one per cent between the 20 and 100 per cent marks of full scale. The service history has been satisfactory. Philip Donely, USA

923. **Paul B. Gooderum, George P. Wood, and Maurice J. Brevoort, "Investigation with an interferometer of the turbulent mixing of a free supersonic jet,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1847, Apr. 1949, 65 pp.

The free turbulent mixing of a supersonic jet of Mach number 1.6 was investigated experimentally with an interferometer, of which a description is given. Density and velocity distributions through the mixing zones were obtained.

In the subsonic portion of the mixing zone, the velocity distribution was found to fit the theoretical distribution for incompressible flow. It was also found that the rates of spread of the mixing zone, both into the jet and into the ambient air, were smaller than those for subsonic jets. Bennett H. Edelman, USA

Thermodynamics

(See also Revs. 880, 899, 920, 936)

924. **Kyrille Popoff, "On the impossibility of reaching zero absolute temperature (Sur l'impossibilité d'atteindre le zéro des températures absolues),"** *C. R. Acad. Sci. Paris*, Mar. 14, 1949, vol. 228, pp. 908–910.

Nernst had proved that absolute zero temperature cannot be reached by any finite process. The author presents a new proof of this proposition. Serge Gratch, USA

Heat Transfer; Diffusion

(See Revs. 847, 906, 952)

Acoustics

(See also Rev. 824)

925. **R. W. Morse, "Dispersion of compressional waves in isotropic rods of rectangular cross section,"** *J. acoust. Soc. Amer.*, Nov. 1948, vol. 20, pp. 833–838.

In this paper the author presents the results of experiments on the dispersion effect of elastic waves in brass rods of rectangular and circular cross sections. After a discussion of the theory of two-dimensional elastic vibrations (plates) a short account of the method of measuring the dispersion curves (d.c.) is given.

The d.c., that is, the phase velocity of the elastic waves plotted against frequency, is found to consist of two branches corresponding to the two velocities of propagation in the directions of the two axes of the rectangular cross section of the bar. One branch of the d.c. approaches the limiting shear velocity in the high-frequency range, whereas in the low-frequency range—as the wave length approaches the value of the width of the bar—the phase velocity increases without limit. On the other hand, the second (low-frequency) branch of the d.c. corresponding to the lower velocity of the waves—belonging to the longer side of the rectangle—approaches the compressional velocity of the rod on the low-frequency side and the shear velocity in the high-frequency range. These two branches coalesce when the cross section becomes a square. Finally it is found that the d.c. for a square cross section coincides with that of a circular cross section if the areas of the sections are equal, although it is not known for certain that this holds in the general case.

Nicholas Chako, USA

926. **Morikazu Toda, "Absorption of sound waves in plastic mediums"** (in Japanese), *Appl. Math. Mech. (Oyō Sūgaku Rikigaku)*, Nov.-Dec. 1947, vol. 1, pp. 253–258.

Discussing the periodic solution of the Boltzmann equation, that is $S(t) = E\gamma(t) - {}_0\int^t f(t-\theta)\gamma(\theta)d\theta$, where S is the stress, E the elastic constant, γ the strain and f the influence function, the author obtains the velocity and the coefficient of absorption of a sound wave for some form of $f(\theta)$. A brief discussion is also given of the case when the medium has some complex properties of relaxation. Takeo Mogami, Japan

Ballistics, Detonics (Explosions)

(See also Revs. 851, 772)

927. **M. R. Rousselet, "The use of Grassot's fluxmeter for measuring initial velocity of projectiles on proving grounds (Emploi du fluxmètre Grassot pour la mesure de la vitesse initiale des projectiles sur les champs de tir),"** *Mémor. Artill. fr.*, 1948, vol. 22, no. 3, pp. 649–655.

Time intervals of order 0.1 sec are measured within about 0.2 per cent. The currents through two orthodox velocity screens are balanced, and a known current flows through the fluxmeter during the period between breaking the first screen and breaking the second. H. H. M. Pike, England

Soil Mechanics, Seepage

(See also Rev. 822)

928. **Gregory P. Tschebotarioff and Edward R. Ward, "The resonance of machine foundations and the soil coefficients which affect it"** (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 1, pp. 309–313.

Formulas for the natural frequency of foundations are reviewed. Vibration results from a compressor foundation are analyzed together with laboratory results and published earthquake data. It is suggested that the natural frequency can be expressed as $\omega = \omega_r/p^{1/2}$ where ω_r is the frequency under unit average ground pressure and p is the average ground pressure.

R. N. Arnold, Scotland

929. **A. Casagrande and W. L. Shannon, "Strength of soils under dynamic loads,"** *Proc. Amer. Soc. civ. Engrs.*, Apr. 1948, vol. 74, pp. 591–608.

Studies on the strength of soils and soft rocks under static and dynamic loading are described. Three types of apparatus to apply

transient loads in triaxial compression and unconfined compression tests have been developed, a pendulum loading type, a falling-beam loading type, and a hydraulic loading type. Furthermore, two types of apparatus to apply static loads were used, the Fairbank-scale loading type and a hydrostatic loading type. The materials tested are: Manchester (N.H.) sand; Cambridge (Mass.) clay; Boston (Mass.) clay; Stockton (Calif.) clay; Atlantic muck (Canal Zone); and Cucaracha shale (Canal Zone).

The findings of this report are summarized as follows: The strength of the clays and of the Cucaracha shale loaded to failure in about 0.02 sec was found to be between 1.5 and 2.0 times greater than their 10-min static strengths. The strength of sand increased only slightly with decreasing time of loading. The modulus of deformation of clay and Cucaracha shale for fast transient tests was found to be approximately twice that for 10-min static tests. The modulus of deformation of sand was found to be independent of the time of loading.

In a discussion of this paper [*Proc. Amer. Soc. civ. Engrs.*, Sept. 1948, pp. 1251–1254] G. P. Tschebotarioff cites experiments made at Princeton University. Some of these tests indicated that sands could be very strongly affected by vibratory loads producing deformations as much as 140 times greater than deformations produced by a single application of an equivalent static load.

R. K. Bernhard, USA

930. Oreste Moretto, "Effect of natural hardening on the unconfined compression strength of remolded clays" (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, June 1948, vol. 1, pp. 137–144.

Experiments were made with four types of clay, a Laurentian clay from Quebec, two types from Detroit and one from Mexico.

Seven series of tests of the Laurentian clay were made at different water contents. Tables and figures of the results are given, and show a tremendous increase in both strength and rigidity, from the time the clay was allowed to rest at constant water content. The sensitivity acquired after 610 days of rest was one third of its value in the natural undisturbed state. The seventh series was for the determination of the relation between the increase in strength and the relative consistency, after a fixed period of rest; it was found that for a given period of rest, the relative increase in strength increases with the relative consistency of the clay, and in the range of consistencies tested, the rate of hardening increases with the water content.

Four groups of tests were made on the first Detroit clay. The results are given in tables and figures, but are not extensive enough to afford a definite conclusion, although they indicate that this clay has its maximum rate of hardening for a water content near the liquid limit.

Only one series of tests was made on the second Detroit clay. This showed that the rate of increase of strength and rigidity for this clay was slower, and after 240 days of rest the stress-strain curve acquired a shape similar to that for the undisturbed state; sensitivity reached about one third that of the undisturbed state.

A short series of tests was made on the Mexican clay, which showed that the increase in strength, during the 60-day duration of this series, was small when compared with the natural sensitivity of this clay. P. J. Vlahakis, USA

931. Harris Epstein, "Reduction of lateral cohesive soil pressure on quaywalls by use of sand dikes" (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 3, pp. 291–296.

An extensive development is given of the subject of the author's paper which was presented at the symposium on lateral earth pressures on flexible retaining walls [*Proc. Amer. Soc. civ. Engrs.*, Jan. 1948, fourth paper; see Rev. **850,** May 1948].

The theoretical relieving effect of sand dikes of different shapes interposed between an anchored bulkhead and a fluid clay backfill (as compared to a backfill composed entirely of sand) is analyzed under the assumption of: (*a*) failure conditions governed by an angle of internal friction of 30 deg; (*b*) varying limit values of the angle of wall frictions; (*c*) plane surfaces of rupture and a uniform distribution of the shearing resistance over the entire surface of rupture. Only active earth pressures against the portion of the bulkhead above the dredge line are considered.

The results of this analysis are compared to the results of large-scale model earth-pressure tests at Princeton University reported by the reviewer in the second paper of the afore-mentioned symposium. In the case of trapezoidally shaped dikes the computed and the experimental values of relative effectiveness agree within 5 per cent, which is the approximate accuracy of the measurements. In the case of rectangular dikes the computed values give an effectiveness of dike action which is approximately 40 per cent higher than the corresponding values of relative effectiveness recorded during the experiments.

In the reviewer's opinion this difference is due to the necessarily simplified assumptions of the theoretical analysis which cannot take into account the effect of the deformations of the bulkhead-sand-fluid clay system on the interaction of pressures between the component parts of the system. Foremost among such effects is the restraint at the dredge-line boundary which influences the sand deformations above it, and which is greater in trapezoidal dikes than in rectangular dikes.

Gregory P. Tschebotarioff, USA

932. H. Jansson, A. Wickert, and A. Rinkert, "Earth pressure against retaining walls" (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 2, pp. 71–76.

In this paper the authors present data obtained from a half-scale model, built by the harbor building department of the Port of Stockholm Authority, and consisting of a reinforced-concrete test wall two meters high and six meters long which was subjected to lateral pressures by a crushed-stone backfill and a surcharge placed thereon. The fill rested on a rigid horizontal surface two meters below the top of the wall. The overturning moment of the wall was measured, and wall deflections were taken at thirteen different points. End effects were minimized by the use of lubricated steel end surfaces.

The wall deflection was about 1/3300 of the height.

In making an analysis of the data the authors assumed triangular distribution of lateral pressure, and estimated the pressure on this basis. Some variation in pressure was observed with different filling procedures, but in general the estimated pressures agreed very well with Rankine values computed from measurements of the angle of repose. C. Martin Duke, USA

933. Helmut Fricke, "Model experiments for investigating the soil pressure in the mining industry (Modellversuche zur Erforschung des Gebirgsdruckes im Bergbau)," *Z. Ver. dtsch. Ing.*, Jan. 1949, vol. 91, pp. 33–36.

The author reviews various German and British model experiments for investigating soil pressure in the mining industry. He presents in particular the results of experiments on plates and beams used for tunnel support, made to find the distribution of the earth stresses during the mining of coal. He considers the effect of leaving pillars standing to be disadvantageous, and describes the methods used by Seidl and Eggert to study their influence. He also mentions researches using photoelasticity for determining

the distribution of stresses near a gallery. There are no original results in the paper. L. J. Tison, Belgium

934. **E. N. Fox, "The mean elastic settlement of a uniformly loaded area at a depth below the ground surface"** (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 1, pp. 129–132.

The author makes reference to Boussinesq's well-known formula for the determination of the elastic settlement of a rectangular surface footing. Further reference is made to a publication of Mindlin ["Force at a point in the interior of a semi-infinite solid," *Physics*, 1936, vol. 7, p. 195], which inspired the author to derive estimates for the elastic settlements of sunk footings by following the same procedure as with Boussinesq's formula for a surface footing. He carried out the necessary analysis and calculations at the Building Research Station prior to the war.

He finally arrives at the result that the ratio of the mean settlement W_c of a footing at depth c to that of a surface footing W_0 is given by the equation:

$$W_c/W_0 = (\Sigma_1^5 \beta_s Y_s)/(\beta_1 + \beta_2)Y_1$$

where the β's denote various functions of Poisson's ratio and the Y's denote logarithmic expressions dependent upon the dimensions and upon the depth of the loaded area.

The merit of the paper lies in the mathematical solution of the problem, and it is not sufficiently connected to practical applications and developed in diagrams. Ch. Széchy, Hungary

935. **V. A. Florin, "The consolidation of an earth mass"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, 1948, vol. 59: Jan. 1, pp. 21–24; Jan. 11, pp. 219–222.

A three-phase earth mass consisting of a liquid, a solid and a gaseous phase is considered, its voids ratio ϵ being a function of the sum of the principal stresses θ. The relative proportion of the three phases ($m + n + s = 1$) is assumed to be the same so far as any part of the volume of the mass and the area of any cross section is concerned. This relative proportion is easily connected with the average moisture content η of the mass.

The author designates by **u**, **v** and **w** what he calls the "seepage velocities" of each of the three phases, respectively, and writes the equations of the discharge of each of the three phases beyond the boundaries of the earth mass. He combines the three equations in question, considering the density of the gas ρ as a function of the gas pressure $p + p_0$ (where p_0 is the atmospheric pressure) after which the following equation is obtained:

$$\text{div } (\mathbf{u} + \mathbf{v} + \mathbf{w}) + (s + \mu m)(\partial p/\partial t)/(p + p_0) + (1/\rho)\ (\mathbf{w}, \text{grad}\rho) = 0,$$

in which μ is the coefficient of solubility of the gaseous phase in the liquid and t is the time.

Further work is carried out using the Darcy formula for the velocity of the liquid phase with respect to the soil skeleton (solid phase). Neglecting small values, the author obtains the basic equation of consolidation in the form

$$(\partial\epsilon/\partial t)/(1 + \epsilon) + \beta(\partial p/\partial t) - \text{div } k \text{ grad } H = 0$$

where k is the average coefficient of permeability of the mass, H is the hydraulic head and β a value to be determined experimentally.

In the second paper the variability of the values k and θ is considered. Two sketches illustrate the numerical solution of a corresponding problem. D. P. Krynine, USA

936. **J. D. Coleman, "Soil thermodynamics and road engineering,"** *Nature Lond.*, Jan. 22, 1949, vol. 163, pp. 143–145.

This paper outlines a fundamental approach to moisture transfer being made at the Road Research Laboratory, Middlesex, England. Presented are theoretical relations between the curvature of the air-water interface, the relative humidity, the capillary potential and the temperature.

Typical curves relating vapor pressure, temperature and moisture content are shown, and it is pointed out that the relative humidity of soil moisture is sensibly different from 100 per cent only at very low moisture contents. It is concluded that: (1) differences in moisture content will cause transfer principally in the liquid phase, due to suction gradients; (2) differences in temperature will cause transfer principally in the vapor phase, due to vapor-pressure gradients.

The heat of wetting is presented as a function of capillary potential and temperature. Edward S. Barber, USA

937. **L. Marivoet, "Control of the stability of a sliding slope in a railway cut near Wetteren"** (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 2, pp. 38–42.

The author reports the results of a stability analysis based on explorations and soil tests on a sliding slope. The analysis showed the slope to be unstable as observed. The trouble was attributed to a high water table in loam overlying stiff clay, and lowering of the table has apparently stopped further movement.

C. Martin Duke, USA

938. **R. Haefeli, "The stability of slopes acted upon by parallel seepage,"** (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 1, pp. 57–62.

The problem studied is: Given an earth slope of infinite height, subjected to a parallel flow in any direction, to find the limiting value of the slope under conditions of unstable equilibrium. It is assumed that the angle of internal friction of the cohesionless material considered equals its angle of repose without seepage. Failure would then occur along a plane parallel to the slope. The same phenomenon would take place in a cohesive material if cohesion were zero at the slope and gradually increased toward the inside, but in the common case of a cohesion constant throughout the mass the theory of the paper would not hold, and curved failure lines would have to be considered. The limiting slope found does not depend on the velocity of flow, except in the case of internal erosion of fine sand particles from a cohesionless material; it does depend on the value of the hydraulic gradient along a normal to the slope. A parallel flow may be either harmful or stabilizing so far as the slope is concerned, according to the direction of the flow.

The paper clearly establishes the difference between equipotential and equipressure lines which are sometimes confused. The paper also makes very accurate use of concepts concerning the theory of seepage and pore pressures. The author's term "parallel flow" means the case when the flow lines are parallel or approximately parallel. Though the author advances an example of a slope destroyed step by step, due only to a small initial slide, his paper is of theoretical rather than practical interest. A similar but less involved problem is discussed on p. 426 of D. W. Taylor's recent book, *Fundamentals of Soil Mechanics*.

D. P. Krynine, USA

939. **Wilson V. Binger and Thomas F. Thompson, "Excavation slopes,"** *Proc. Amer. Soc. civ. Engrs.*, Apr. 1948, vol. 74, pp. 570–590.

The authors stress the importance of thorough study of the

proposed sea-level Panama Canal project from the standpoint of engineering geology and soil mechanics, because of the magnitude of the project and the landslides which have occurred previously in the Canal, both during and after construction. A comprehensive history of the Culebra and Cucaracha slides is given, along with a detailed tabulation of the formations encountered and the method of calculating the critical slope by combining laboratory results with studies of previous failures. A discussion of this article in the September issue, *Proc. Amer. Soc. civ. Engrs.*, includes an interesting explanation of the weakness of the Culebra formation by Justin. Robert Quintal, Canada

940. Édouard Lefranc, "Measurement of the permeability of soils in situ and its applications (Mesure de la perméabilité des sols en place et ses applications)," *Génie civ.*, Aug. 15, 1948, vol. 75, pp. 304–308.

Reference is made to the method previously described by the author [*Génie civ.*, Nov. 13, 1937] of measuring the permeability of ground in situ by observing the discharge Q from a well tube in which the water is maintained at a constant head H above ground water level. At the bottom of the tube a roughly spherical hole is formed of radius r (approximately equal to the radius of the tube) and the coefficient of permeability k is given by the equation

$$k = Q/(4\pi\, r\, H).$$

The paper concludes with a brief discussion of the practical applications of a knowledge of permeability.

A. W. Skempton, England

941. J. J. Kolhuszewski, "An experimental study of the maximum and minimum porosities of sands" (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 1, pp. 158–165.

The purpose of this study is to establish indexes of porosity for sandy soils such as Atterberg's limits (plasticity and flow indexes) afford for clayey soils. The porosity of a given sand in situ conveys little information in itself, but its ratio to the maximum and minimum porosities of the material would have much more meaning.

As a result of a carefully performed series of tests in which the disturbing influence of all factors are discussed, the author suggests the following standard procedures for the determination of the limiting porosities of sands: (1) Maximum porosity: 1000 grams of dry and thoroughly mixed sand are placed in a 2000-cc glass cylinder. The cylinder with the sample is shaken a few times, turned upside down and then very quickly turned over again. The volume of the sample is then read and the (maximum) porosity calculated. (2) Minimum porosity: A proctor-type compaction cylinder is placed in a water tank and a sand sample is vibrated in it in three layers (15 min for each layer) using a pneumatic or electric hammer ("Kango hammer"). A $^1/_{30}$-cu ft sample is obtained and the (minimum) porosity calculated. The author suggests that the porosity of sand found in the field be expressed in terms of these maximum and minimum porosities and be called the "relative porosity."

Ch. Széchy, Hungary

942. Kano Hoshino, "A practical method of rapid measurement of soil moisture and its applications" (in English), *Proc. Sec. int. Conf. Soil. Mech. Found. Engng.*, 1948, vol. 1, pp. 270–274.

The author's method consists in determining the density of a specimen by measuring the volume of water displaced by a given weight of soil. It is assumed that the true density of the solid particles is a constant, and consequently that the moisture content is a linear function of the specific volume. A simple apparatus of standard flask, funnel, etc., facilitates routine measurements.

The method, which avoids the delay involved in passing a sample through the drying oven, is of particular value for making relative measurements, and for comparing the moisture contents at different points of a homogeneous embankment. Some examples are given, of both old and recent embankments. The apparent density was measured in the field by taking a sample of a given volume with the aid of a hollow metal cylinder, the sample being leveled off at the bases with a knife.

Jean Goguel, France

943. A. Mortensen, "Field tests with new combined loading test sampler for harbour extension work" (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 1, pp. 244–248.

A brief description is given of a piston sampling tube with which loading tests can be carried out at the bottom of a borehole, prior to taking a core. This apparatus has been used at Aalborg Harbor in late-glacial Yoldia clay and a chalky boulder clay. Laboratory tests were carried out on samples of these clays. It was found that the laboratory strength and the in situ bearing capacity both increase with depth below the surface of the stratum; however, the relationship between the two measurements is not discussed in the paper. The technique may prove useful under conditions where reliable samples cannot be obtained.

A. W. Skempton, England

944. I. Evans and G. G. Sherratt, "A simple and convenient instrument for measuring the shearing resistance of clay soils," *J. sci. Instrum.*, Dec. 1948, vol. 25, pp. 411–414.

The instrument described, known as a "vane," works on the following principle: a system of blades or vanes at the end of a rod and with their planes passing through the axis of the rod is inserted into the soil and the torque required to cause the soil to fail under the stresses exerted by the vanes is measured. For soils of high clay content a linear relationship is found between torque and compressive strength. A simple theory is given for the instrument, relating the torque with the cohesion and angle of internal friction. The theory is in good agreement with experiments for angles up to at least 10 deg. The effect on the torque of the rate of rotation of the vanes has been investigated, and it is shown that the behavior of the vane is analogous in some respects to that of a Couette viscometer.

The velocity of applying the torque must not exceed a certain limit and the depth below the surface also has considerable effect. The instrument was developed by the British Army Operational Research group and has been successfully applied in cross-country field investigation work. Ch. Széchy, Hungary

945. Lyman Carlson, "Determination in situ of the shear strength of undisturbed clay by means of a rotating auger" (in English), *Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 1, pp. 265–270.

In soil mechanics one of the most important problems is the determination of the shear strength of clay strata. For clays of low or moderate sensitivity a procedure for taking relatively undisturbed samples and testing them in the laboratory has been evolved and found by experience to give reliable results. For clays of high sensitivity and especially in those cases where the stratum extends to depths of more than 30 or 40 ft this procedure is not always reliable.

In this paper a method of measuring the shear strength of clays

in situ is described, and two examples are cited in which the strength so measured was in agreement with the strength calculated from deep slips, although the strength as measured on samples was only about one half of the correct values. This represents a real advance in soil-mechanics technique.

The strength is measured in situ by finding the torque required to cause rotation of a small four-bladed vane. The strength is taken as the cohesive force developed over the periphery of a cylinder of height and diameter equal to that of the vane. The vane itself is fixed to the end of a rod which extends up to the surface and which is protected by a casing. The apparatus is pushed or driven into the ground. At any required depth the vane is advanced beyond the casing and a test is carried out. The whole is then pushed to a greater depth and a further test is made; the process is repeated until the full depth of the clay has been traversed. As the depth is increased, the casing and torsion rod are lengthened by extensions.

No dimensions of the apparatus are given, and the mechanical details are still in the process of development, but it is understood that a full report will be published by the Royal Swedish Geotechnical Institute. A. W. Skempton, England

946. **A. W. Skempton, "Vane tests in the alluvial plain of the River Forth near Grangemouth,"** *Géotechnique Lond.*, Dec. 1948, vol. 1, pp. 111–124.

The author gives an account of clay shear tests in situ using an apparatus similar in principle to the rotating-auger device used in Sweden [*Proc. Sec. int. Conf. Soil Mech. Found. Engng.*, 1948, vol. 1, p. 265]. It consists of four vanes rigidly attached to a shaft which is twisted in the soil until failure occurs. The applied torque is measured, and the shear strength of the soil computed. This was done in a clay soil for both the undisturbed and the disturbed state and the values of cohesion computed. These values were then compared with values obtained from corresponding laboratory compression tests. Good agreement was found to exist down to a depth of 45 ft for the undisturbed clay. From this level down to 100 ft (the limit of the borings) the vane tests gave increasingly greater values. The results from the remolded clay were in close agreement throughout the entire depth.

In discussing the strength profile in recent clays the author suggests that the ratio of the cohesion to the overburden pressure for normally loaded clays might be expressed as a function of the liquid limit, in the lower layers. Whereas it is quite apparent that the increase in strength near the surface is due to drying-out, it is not at all clear what produces the relatively high strength in the layer directly below, where the drying effect is questionable. The tests described in this paper are indeed very revealing and clearly indicate the need for further investigations to enable us to appraise more accurately the value of laboratory test results.

Eben Vey, USA

Geophysics, Meteorology, Oceanography

947. **M. Magata, "On a method of integration of the differential equations of motion in dynamical meteorology"** (in Japanese), *J. met. Soc. Japan*, July 1948, vol. 26, pp. 29–32.

The equations of horizontal motion of the ideal atmosphere, in Eulerian form, are quasi-linear partial differential equations, so that by using Cooley's method they can be reduced to Hamiltonian form:

$$dx/dt = \partial H/\partial p_x,\ dp_x/dt = -\partial H/\partial x,\ dy/dt = \partial H/\partial p_y,\ dp_y/dt = -\partial H/\partial y,$$

where $p_x = u - \frac{1}{2}\lambda_y$, $p_y = v + \frac{1}{2}\lambda x$, $H = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(p_x y - p_y x) + \frac{1}{8}\lambda^2(x^2+y^2) + p/\rho$.

When the pressure p is a function of r (radial distance) or $ax + by$ only, the general solution is obtained by applying a theorem of analytical dynamics. For the former case, the solution is given by $\frac{1}{2}(u^2 + v^2) + p(r)/\rho = h, vx - uy + \frac{1}{2}\lambda r^2 = c$, where h and c are arbitrary constants. By applying this to the problem of a typhoon, the mechanism of generation of the eye of a storm is interpreted. Sigekata Syōno, Japan

948. **A. F. Crossley, "On the relation between wind and pressure,"** *Quart. J. roy. met. Soc.*, July-Oct. 1948, vol. 74, pp. 379–382.

The equations for steady horizontal motion of the atmosphere are solved for the case of zonally symmetric flow at low latitudes. It is shown that specifying the pressure field "is not sufficient to determine the horizontal motion of air. . . . Some further initial condition is required." M. Neiburger, USA

949. **Sigekata Syōno, "On the mechanism of generation of cold waves"** (in Japanese), *J. met. Soc. Japan*, July 1948, vol. 26, pp. 1–7.

The mechanism of the generation of cold waves is treated mathematically. The author indicates that when the pressure gradient and tangential wind velocity are lower than their critical values the cold anticyclone is in stable equilibrium, while when they surpass the critical values the equilibrium is lost, giving rise to radial acceleration. K. Terada, Japan

950. **Sigekata Syōno, "On the equivalence of the 'slice method' and the 'parcel method'"** (in Japanese), *J. met. Soc. Japan*, July 1948, vol. 26, pp. 63–65.

For the analysis of vertical stability the "slice method" and the "parcel method" are generally used. Though seemingly different, their physical nature is the same. In this paper the equivalence of the two methods is discussed, and the process of application of the slice method to the adiabatic chart is studied.

K. Terada, Japan

951. **Giiti Yamamoto and Masao Siotani, "Turbulence in the free atmosphere"** (in Japanese), *J. met. Soc. Japan*, Oct.-Dec. 1947, vol. 25, pp. 120–132; June 1948, vol. 26, pp. 149–157.

Continuous recording of the wind speed was made by five hot-wire anemometers mounted on a steel tower of Maebasi broadcasting station, the heights of the anemometers being 38, 41, 44, 47 and 50 meters above the ground. Taking means over a time interval of 5 min, the authors determine the mean speed, root-mean-square fluctuation, time-and-space correlation coefficients of fluctuations, coefficient of eddy viscosity, etc. The structure of natural wind is discussed in terms of these statistical characteristics and compared with the corresponding data on the flow in wind tunnels.

The second report is concerned with the effect of reducing the time interval over which the means are taken. It is shown that reduction of the time interval reduces the root-mean-square fluctuation, the width of correlation curve and the coefficient of eddy viscosity. Itiro Tani, Japan

952. **Heinz Lettau, "On the theory of partial separation of gases in the atmosphere (Zur Theorie der partiellen Gasentmischung in der Atmosphäre),"** *Met. Rdsch.*, 1947, vol. 1: July-Aug., pp. 5–10; Sept.-Oct., pp. 65–74.

The author introduces the "factor of unmixing"

$$Q = \mu/(\mu + \Lambda)$$

(where μ is the coefficient of viscosity and A the exchange coefficient) as a simple means of defining the degree of mixture homogeneity of every atmospheric layer. Q must lie between 0 and 1, with $Q = 0$ representing a maximum degree of mixing, and $Q = 1$ representing a pure gravity diffusion equilibrium. Hitherto, only the extreme conditions represented by $Q = 1$ or $Q = 0$ have been taken under consideration.

In a real atmosphere, the exchange coefficient is never zero or infinite. Every steady state can be defined by means of the "unmixing factor" and its variation with height. Therefore, it is possible to find A or $A(z)$ by means of chemical analysis in the upper atmosphere, using the equation

$$s = s_0 \exp(-pH^{-1} \int_0^z Q dz)$$

[where s is the mass concentration of a component of the atmosphere, H the height of the homogeneous atmosphere (7.99 km), m_s the molecular weight of the component, m the molecular weight of the air, and $p = (m - m_s)/m$] and the equation

$$A(z) = \mu[1/Q(z) - 1].$$

Gases which enter the atmosphere at one level and leave at another have a certain rate of vertical flow for each intermediate layer. It is possible to compute this rate of flow for various heights. Examples of such gases are helium and hydrogen, and, for the troposphere only, ozone.

If a gas has a constant "basic flow" S_0, the theory works with the apparent factor of unmixing q instead of Q. If S_0 has the critical value of $-\mu s_0 p/H$, then the concentration of a light gas does not increase with height even in the case of gravity-diffusion equilibrium.

In nonsteady cases the change of composition of the atmosphere as a function of the changes of Q is best expressed by a Bessel equation. A calculation is made of the time needed for the transformation of exchange equilibrium into gravity-diffusion equilibrium, and for the reverse transformation. For the first case some 1000 yr are required, for the reverse case weeks to months are required if $A = 50$ g per cm sec.

The medium Q-value for the troposphere is 0.000004, too small to result in unmixing and permitting only gases with a basic flow to show a change in concentration with altitude.

In the stratosphere the oxygen analyses of Regener give: $Q = 0.11(z - 14)/H$; $z > 14$ km; and the stratospheric law of exchange variation with height becomes

$$A(z) = \mu[H/0.11(z - 14) - 1]$$

for $14 < z < 28$ km. For heights of 15 km, 20 km, and 28 km this gives A-values of 0.01, 0.0002, and 0.0008 g per cm sec, respectively.

From the He-values of Paneth one gets: $q = 0.015(z - 14)/H$. The difference between Q and q for He permits the calculation of the basic flow of He as follows: $S_0 = -s_0\mu(p/H)(Q - q)/q = 3.4$ m³ per sec for the total earth surface.

The stratospheric law of exchange partially explains the sharp increase of ozone at altitudes of 14 to 15 km.

Hans F. Winterkorn, USA

953. Walter H. Munk, Hector V. Iglesias, and T. R. Folsom, "An instrument for recording ultra low frequency ocean waves," *Rev. sci. Instrum.*, Oct. 1948, vol. 19, pp. 654–658.

Instruments for recording ocean swell waves (with periods up to 30 sec) and tides are well-known. The authors now describe an instrument for wave measurement in the intermediate range of periods. The hydraulic system suppresses all very low and very high frequencies by means of capillaries and air chambers—comparable with filters in an electric R-C circuit. Maximum response is given for waves of 15-min period, the period of the band pass extending from about 2 min to 2 hr. Records obtained show the continual presence of waves in this yet unexplored part of ocean-wave frequencies. Horst Merbt, England

Lubrication; Bearings; Wear

954. J. A. Haringx, "The cross-spring pivot as a constructional element," *Appl. sci. Res. Sect. A*, 1949, vol. 1, no. 4, pp. 313–332.

This pivot consists of two or more spring leaves joining two rigid members. It forms a closed pair permitting limited relative rotation. In instrument and machinery applications, its advantages are claimed to be the absence of sliding parts, immunity to dirt, and desirable elastic properties. The author calculates the response of this pivot to a general plane force system for small deflections and to a pure moment for large deflections. Correlation with experimental data by W. E. Young is made.

G. A. Nothmann, USA

955. L. D. Thompson, S. J. Backey, and E. L. Conn, "Engine wear research," *Soc. auto. Engrs. quart. Trans.*, Jan. 1949, vol. 3, pp. 41–50.

The outstanding lesson of this paper appears to be the fact that wear rate tests of cylinder bore and compression ring wear have been devised which can be repeated. Two engines were used, operating at quite different brake mean effective pressures and speed. An interesting correlation resulting from the test is that the rate of wear appeared to vary directly with the amount of sulphur which went into the engine in the fuel oil, and did not vary particularly with the operating conditions. A theory is presented as to the reason for the variation of wear rate with fuel sulphur content. Erle I. Shobert, II, USA

Marine Engineering Problems

956. John R. Dawson, Robert C. Walter, and Elizabeth S. Hay, "Tank tests to determine the effect on planing-tail hulls of varying length, width, and plan-form taper of afterbody," *Nat. adv. Comm. Aero. Rep.*, no. 844, 1946 (publ. in 1949), pp. 1–8.

NACA has developed a flying-boat hull with a pointed-step forebody in combination with a very long afterbody which extends back to the region where the tail surfaces would be attached. This so-called planing-tail hull is reported to have much lower resistance than conventional hulls.

The present tests indicate that the resistance can be diminished by increasing either the width, or the length, or the plan form taper of the afterbody of a planing-tail hull. The effects of these parameters on the trimming moment required to obtain best trim, and the locations of the center of gravity required to obtain best trim at the hump speed, are also presented.

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